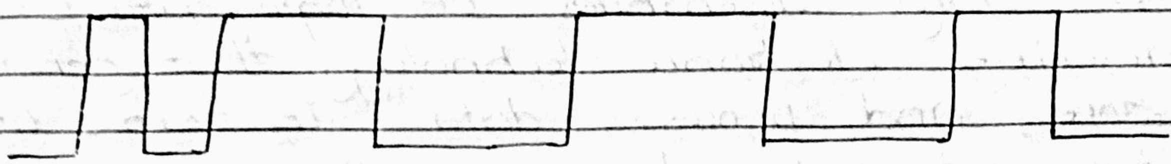
# Digital Electronics

## Digital Signal Representation

Digital Data 0 1 0 1 1 0 0 1 1 0 0 1 0



Digital Signal

Digital signal is represented ~~by~~ in the form of square wave whereas analog is in the form of sine wave
Sine waves are continous in nature and square are discrete.

## What is Digital Electronics?

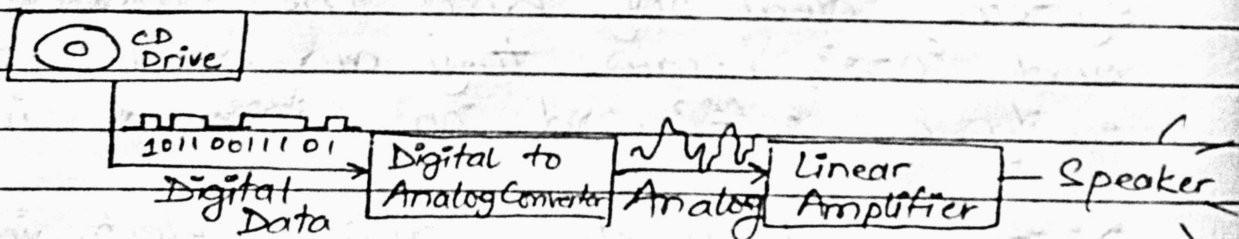
- Digital Electronics deals with the electronic manipulation of numbers, or with the manipulation of varying quantities by means of numbers. Because it is convenient to do so, today's digital systems deal only with the numbers 'zero' and 'one', because they can be represented easily by 'off' and 'on' within a circuit.

- Digital stand for digit, digital electronics basically has two conditions which are possible, 0 (low logic) and 1 (high logic). Digital electronic systems use a digital signal that are composed of mathemiatical features to work.

- 1 as true and 0 as false are called bit and the group of bits are named byte.

- Digital electronic circuits are usually made from large assemblies of logic gates. Digital describes electronic technology that generates, stores, and processes data in terms of two states : 1 and number 0.

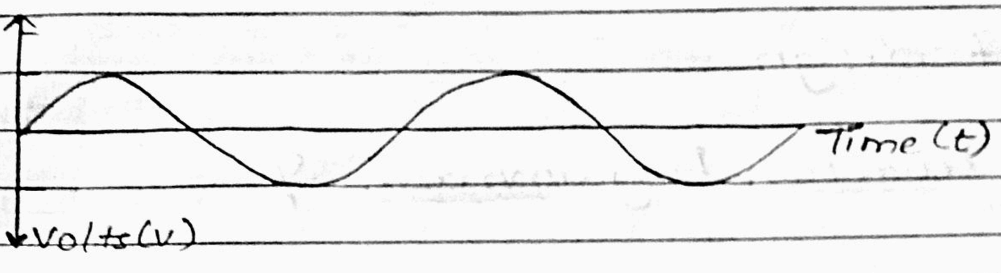
Analog vs Digital
Many systems use a maximum of analog and digital electronics to take advantage of each technology.
Example: A typical CD player accepts digital data from the CD drive and converts its to an analog signal for amplification. Digital data CD drive 1011001101 Analog reproduction of music audio signal speaker sound waves digital-to-analog converter linear amplifier



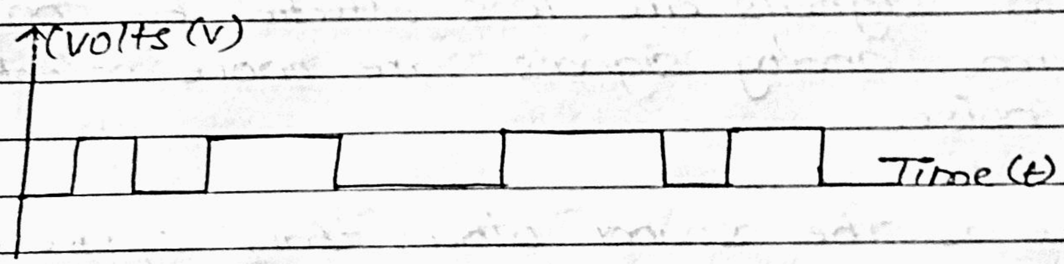

⇒ Digital data is in the form of 0 and 1 and can be understood by computer only whereas analog data cannot ~~by~~ be understood by ~~ka~~ computer.

Amplifier → to increase the strength of signal.

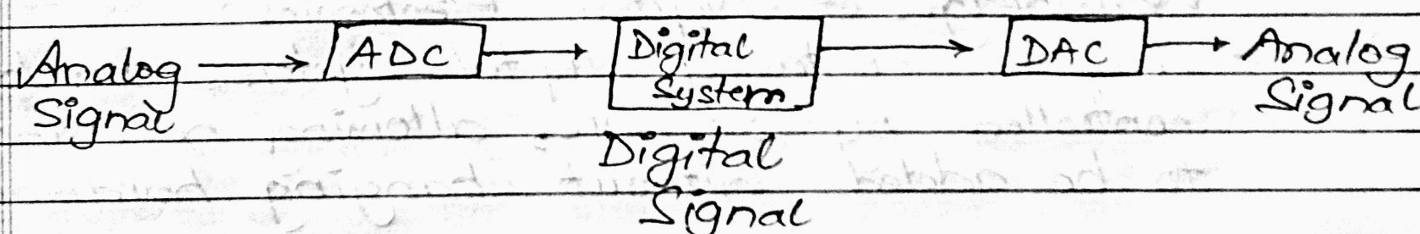
Analog signal representation




Digital signal representation




Conversion of Analog to Digital Signal




There are two types of converters:-

1. ADC → Analog to Digital Converter
2. DAC → Digital to Analog converter

Benefits of Digital over Analog

- Reproductibility
- Not effected by noise means quality
- Ease of design
- Data Protection Programmable Speed
- Economy

~~=> Data~~

=> Digital signals are less affected by noise whereas analog signals are more affected by noise.

=> Noise is the sound other than square or sine waves.

Advantages of Digital Electronics

- Computer controlled digital systems can be controlled by software, allowing new function to be added without changing hardware
- Information storage can be easier in digital systems than in analog ones.
- The noise immunity of digital systems permits data to be stored and retrieved without noise.

- In a digital system are easier to design and more precise representation of a signal can be obtained by using more binary digits to represent it.

- More digit circuitary can be fabricated on IC chips.

- Error management method can be inserted into the signal path. To detect errors, and then correct the errors, or at least ask for a new copy of the data.

Disadvantages of Digital Electronics

- Conversion to digital format and re-conversion to analog format is needed, which always include the lost of information.

- In some cases, digital circuits use more energy than analog circuits and produce more heat and need heat sinks.

- Digital circuits are sometimes more expensive, especially in small quantities.

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Applications of Digital Electronics

It's application are infinite, ~~ragging~~ ranging for high end computing to miniature circuits that can be very versatile, signal processing, communication, etc. Digital electronics is currently rapidlly developing and removing conventional analogue machines due to its high speed, more accurary and greater flexibility.

* The digital system send the data in the form of packets of digital codes, thus we can encode and decode them in various formats and codes.

* Data encryption is also possible in the digital systems, hence data transmission is more secure and can be manipulated in many formats.

* Digital systems are much advantageous in communications Data Transmission using Digital Systems

# Number System

- Decimal Numbers → Base 10 (0 – 9)
- Binary Numbers → Base 2 (0, 1)
- Octal Numbers → Base 8 (0 – 7)
- Hexadecimal Numbers → Base 16 (0-9, A – F)

| Number system | Base (Redix) | Digits | Example |
|---|---|---|---|
| Binary | 2 | 0, 1 | $(1011.11)_2$ |
| Decimal | 10 | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 | $(3567.29)_{10}$ |
| Octal | 8 | 0, 1, 2, 3, 4, 5, 6, 7 | $(356.11)_8$ |
| Hexadecimal | 16 | (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (A=10, B=11, C=12, D=13, E=14, F=15) | $(ABC.13)_{16}$ |

Conversions

$\Rightarrow (74)_{10} \longrightarrow (\quad)_2$

| | | |
|---|---|---|
| 2 | 74 | |
| 2 | 37 | 0 |
| 2 | 18 | 1 |
| 2 | 9 | 0 |
| 2 | 4 | 1 |
| 2 | 2 | 0 |
| 2 | 1 | 0 |
| | 0 | 1 |

$(74)_{10} \longrightarrow (1001010)_2$

$\Rightarrow (74.04)_{10} \longrightarrow (\quad)_2$

| | | |
|---|---|---|
| 2 | 74 | |
| 2 | 37 | 0 |
| 2 | 18 | 1 |
| 2 | 9 | 0 |
| 2 | 4 | 1 |
| 2 | 2 | 0 |
| 2 | 1 | 0 |
| | 0 | 1 |

| | |
|---|---|
| $0.4 \times 2 = 0.08$ | 0 |
| $0.08 \times 2 = 0.16$ | 0 |
| $0.16 \times 2 = 0.32$ | 0 |

$(74.04)_{10} \longrightarrow (1001010.000)_2$

$\Rightarrow (242.4)_{10} \longrightarrow (\quad)_2$

| | |
|---|---|
| 2 | 242 |
| 2 | 121 — 0 |
| 2 | 60 — 1 |
| 2 | 30 — 0 |
| 2 | 15 — 0 |
| 2 | 7 — 1 |
| 2 | 3 — 1 |
| 2 | 1 — 1 |
| | 0 — 1 |

| | |
|---|---|
| $0.4 \times 2 = 0.8$ | 0 |
| $0.8 \times 2 = 1.6$ | 1 |
| $0.6 \times 2 = 1.2$ | 1 |

$(242.4)_{10} \longrightarrow (11110010.011)_2$

$\Rightarrow (73.04)_{10} \longrightarrow (\quad)_8$

| | |
|---|---|
| 8 | 73 |
| 8 | 9 — 1 |
| 8 | 1 — 1 |
| | 0 — 1 |

| | |
|---|---|
| $0.04 \times 8 = 0.32$ | 0 |
| $0.32 \times 8 = 2.56$ | 2 |
| $0.56 \times 8 = 4.48$ | 4 |

$(73.04)_{10} \longrightarrow (111.024)_8$

$\Rightarrow (73.04)_{10} \longrightarrow (\quad)_{16}$

| | |
|---|---|
| 16 | 73 |
| 16 | 4 — 9 |
| | 0 — 4 |

| | |
|---|---|
| $0.04 \times 16 = 0.64$ | 0 |
| $0.64 \times 16 = 10.24$ | $10 \rightarrow A$ |
| $0.24 \times 16 = 3.84$ | 3 |

$(73.04)_{10} \longrightarrow (49.0A3)_{16}$

$\Rightarrow (743.02)_{10} \longrightarrow (\quad)_2$

| | |
|---|---|
| 2 | 743 |
| 2 | 371 − 1 |
| 2 | 185 − 1 |
| 2 | 92 − 1 |
| 2 | 46 − 0 |
| 2 | 23 − 0 |
| 2 | 11 − 1 |
| 2 | 5 − 1 |
| 2 | 2 − 1 |
| 2 | 1 − 0 |
| | 0 − 1 |

$0.02 \times 2 = 0.04$ | 0
$0.04 \times 2 = 0.08$ | 0
$0.08 \times 2 = 0.16$ | 0
$0.16 \times 2 = 0.32$ | 0

$(743.02)_{10} \longrightarrow (1011100111.000)_2$

$\Rightarrow (243.1)_{10} \longrightarrow (\quad)_2$

| | |
|---|---|
| 2 | 243 |
| 2 | 121 − 1 |
| 2 | 60 − 1 |
| 2 | 30 − 0 |
| 2 | 15 − 0 |
| 2 | 7 − 1 |
| 2 | 3 − 1 |
| 2 | 1 − 1 |
| | 0 − 1 |

$0.1 \times 2 = 0.2$ | 0
$0.2 \times 2 = 0.4$ | 0
$0.4 \times 2 = 0.8$ | 0

$(243.1)_{10} \longrightarrow (11110011.000)_2$

$\Rightarrow (742.04)_{10} \longrightarrow (\quad)_8$

| | |
|---|---|
| 8 | 742 |
| 8 | 92 — 6 |
| 8 | 11 — 4 |
| 8 | 1 — 3 |
| | 0 — 1 |

$\cancel{0.4 \times 8 = 3.2}$ | $\cancel{3}$

$\cancel{0.2 \times 8 = 1.6}$

| | |
|---|---|
| $0.04 \times 8 = 0.32$ | 0 |
| $0.32 \times 8 = 2.56$ | 2 |
| $0.56 \times 8 = 4.48$ | 4 |

$(742.04)_{10} \longrightarrow (1346.024)_8$

$\Rightarrow (745)_{10} \longrightarrow (\quad)_{16}$

| | |
|---|---|
| 16 | 745 |
| 16 | 46 — 9 |
| 16 | 2 — 14 |
| | 0 — 2 |

$(745)_{10} \longrightarrow (2E9)_{16}$

⇒ $(748.3)_{10} \longrightarrow (\quad)_{16}$

| | | |
|---|---|---|
| 16 | 748 | |
| 16 | 46 | — 12 |
| 16 | 2 | — 14 |
| | 0 | — 2 |

| | |
|---|---|
| $0.3 \times 16 = 4.8$ | 4 |
| $0.8 \times 16 = 12.8$ | 12 |

$(748.3)_{10} \longrightarrow (2EC.4C)_{16}$

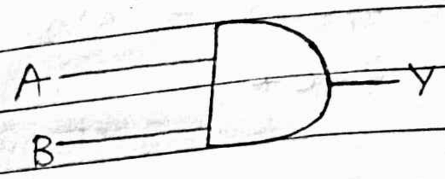
## Logic Gates and Boolean Algebra

### Logic Gates

- Basic Logic Gates
  - AND Gate
  - NOT Gate
  - OR Gate
- Universal Logic Gates
  - NAND Gate
  - NOR Gate
- Other Logic Gates
  - EX-OR Gate
  - EX-NOR Gate

⇒ NAND Gate and NOR Gate are called universal logic gates because any gate can be made with the help of these two logic gates.

AND Logic Gate




integrated circuit

IC no. for AND Gate

↓

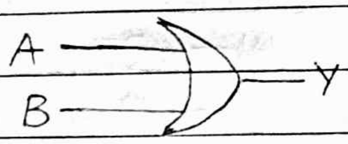
7408

Boolean expression : $Y = A.B$

Truth Table ↓

| A(input) | B(input) | Y(output) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

OR Logic Gate




IC no. for OR Gate

↓

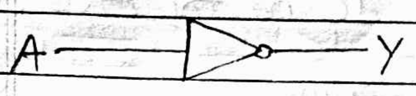
7432

Boolean Expression $Y = A + B$

Truth Table ↓

| A(input) | B(input) | Y (output) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

NOT Logic Gate



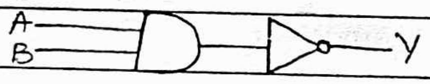

IC no. for NOT Gate
7404

Boolean expression $Y' = \overline{A}$

Truth Table

| A (input) | Y (output) |
|---|---|
| 0 | 1 |
| 1 | 0 |

NAND Logic Gate





IC no. for NAND Gate
7400

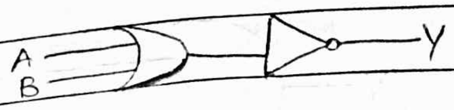
Boolean expression $Y = \overline{A.B}$

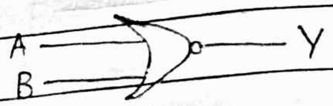
Truth Table

| A (input) | B (input) | Y (output) |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

NOR Logic Gates

IC no. for NOR Gate
↓
7402



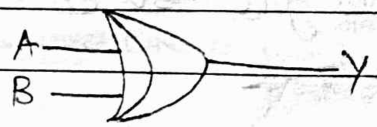

Boolean expression $Y = \overline{A+B}$

Truth Table ↓

| A(input) | B(input) | Y(output) |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

EX-OR Logic Gate

IC no. for XOR Gate
↓
7486

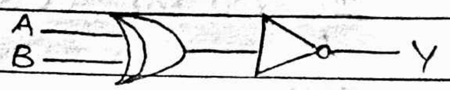


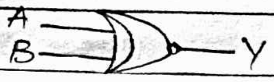

Boolean Expression $Y = A \oplus B \rightarrow \bar{A}B + A\bar{B}$

Truth Table ↓

| A(input) | B(input) | Y(output) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

EX-NOR Logic Gate




Boolean expression $Y = \overline{A \oplus B} = \overline{\bar{A}B + A\bar{B}}$

Truth Table :-

| A(input) | B(input) | Y(output) |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

⇒ IC of all the gates are 14 pins IC
To activate any IC, you need to give +5V on $14^{th}$ pin and 0V on $7^{th}$ pin
0V ↓ Ground

⇒ In one IC of AND Gate, there are 4 AND Gates
We can calculate this as :-
Total pins = 14
Pins for power supply = 2
Pins left = 12
Pins in one AND Gate = 3
Total AND Gates in 1 IC = 12/3 = 4

⇒ Output ~~for~~ will be on 3, 6, 8, 11 for OR, AND, NAND, XOR, XNOR. Gate.
⇒ Output will be on 2, 4, 6, 8, 10, 12 for NOT Gate.

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$\Rightarrow$ Output will be on 1, 4, 10, 13 for NOR Gate

Conversions :-

$\Rightarrow (010.10)_2 \longrightarrow (\quad)_{10}$

$$\begin{matrix} 0 & 1 & 0 & . & 1 & 0 \\ 2^2 & 2^1 & 2^0 & & 2^{-1} & 2^{-2} \end{matrix}$$

$0\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1} + 0\times2^{-2} = 0+2+0+0.5+0$
$= 2.5$

$(010.10)_2 \longrightarrow (2.5)_{10}$

$\Rightarrow (0100.101)_2 \longrightarrow (\quad)_{10}$

$$\begin{matrix} 0 & 1 & 0 & 0 & . & 1 & 0 & 1 \\ 2^3 & 2^2 & 2^1 & 2^0 & & 2^{-1} & 2^{-2} & 2^{-3} \end{matrix}$$

$4+0.5+0.125 = 4.625$

$(0100.101)_2 \longrightarrow (4.625)_{10}$

$\Rightarrow (0100.100)_2 \longrightarrow (4.5)_{10}$

$\Rightarrow (0100.100)_8 \longrightarrow (\quad)_{10}$

$$\begin{matrix} 0 & 1 & 0 & 0 & . & 1 & 0 & 0 \\ 8^3 & 8^2 & 8^1 & 8^0 & & 8^{-1} & 8^{-2} & 8^{-3} \end{matrix}$$

$64+0.125 = 16.125$

$(0100.100)_8 \longrightarrow (64.125)_{10}$

$\Rightarrow (01010.101)_2 \longrightarrow (\quad)_{10}$

$$\begin{array}{ccccccccc} 0 & 1 & 0 & 1 & 0 & . & 1 & 0 & 1 \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & & 2^{-1} & 2^{-2} & 2^{-3} \end{array}$$

$8+2+0.5+0.125 = 10.625$

$(01010.101)_2 \longrightarrow (10.625)_{10}$

$\Rightarrow (467.1)_8 \longrightarrow (\quad)_2$

8421
0110
100
110

$(467.1)_8 \longrightarrow (100110111.001)_2$

$\Rightarrow (467.1)_{16} \longrightarrow (\quad)_2$

$(467.1)_{16} \longrightarrow (010001100111.0001)_2$

$\Rightarrow (010010110.101011)_2 \longrightarrow (\quad)_8$

8421
010
110
101
011

$(010010110.101011)_2 \longrightarrow (226.53)_8$

$\Rightarrow (010010110.10101100)_2 \longrightarrow (\quad)_{16}$

8421
1001
0110
1010
1100

$(010010110.101011)_2 \longrightarrow (96.AC)_{16}$

$\Rightarrow (010010101.00101100)_2 \longrightarrow (\quad)_{16}$

8421
1001
0101
0010
1100

$(010010101.001011)_2 \longrightarrow (95.2C)_{16}$

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$\Rightarrow (74A.A5)_{16} \rightarrow (\quad)_8$

$(74A.A5)_{16} \rightarrow (011101001010.10100101)_2$

$(011101001010.10100101)_2 \rightarrow (3512.512)_8$

8421
011
101
001
010
101
001
010

$\Rightarrow (742.10)_8 \rightarrow (\quad)_{16}$

8421

$(742.10)_8 \rightarrow (111100010.001000)_2$

$(111100010.0010)_2 \rightarrow (1E2.20)_{16}$

$\Rightarrow (ABC.9A)_{16} \rightarrow (\quad)_8$

$(ABC.9A)_{16} \rightarrow (101010111100.10011010)_2$

$(101010111100.10011010)_2 \rightarrow (5274.464)_8$

8421
101
010
111
100
110
100

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# Binary Arithmetic

## Binary Addition:

| A | B | sum | carry |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

```
  1 1  1
 011010
+001011
 100101
```

```
  1111
  1111
+ 1011
 11010
```

```
 1111
 10111
+01101
100100
```

Binary subtraction ↓

| A | B | difference | borrow |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

```
  10011
- 01100
   0111
```

```
  1010011
- 0100100
  0101111
```

```
  0110011
- 0010101
  0011010
```

```
  1001010
- 0011001
  0110001
```

```
  100001
- 000010
  011111
```

Binary Multiplication →

1001
× 10
0000
1001×
10010

1111
×1111
1111
1111×
1111××
1111×××
11100001

1011
×0111
1011
1011×
1011××
0000×××
1001101

1010011
− 0100100
0101111

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Boolean Algebra

⇒ Boolean Algebra is basically used to reduce the expression

Laws of Boolean Algebra

NOT $\bar{\bar{A}} = A$

AND $A.\bar{A} = 0$

$A.A = A$

$A.0 = 0$

OR $A+0 = A$ $A+\bar{A} = 1$

$A+1 = 1$

$A+A+A = A$

$AB + A\bar{B}$

$= A(B+\bar{B}) = A$ $\because B+\bar{B} = 1$

$A\bar{B} + AB\bar{C} + ABC$

$A\bar{B} + AB(\bar{C}+C)$ $\because C+\bar{C} = 1$

$A\bar{B} + AB$

$= A(\bar{B}+B)$ $\because B+\bar{B} = 1$

$= A$

$(A+B)(A+C)$

$AA + AC + BA + BC$ $\because A.A = A$

$A + AC + AB + BC$

$A(1+C) + AB + BC$ $\because 1+C = 1$

$A + AB + BC$

$A(1+B) + BC$ $\quad \because 1+B = 1$

$= A + BC$

⇒ Distributive Theorem

$(A+B)(A+C) = A + BC$

$(A+B)(A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B})$

$(A+B\bar{B})(\bar{A}+B\bar{B})$ $\quad \because B\bar{B} = 0$

$A.\bar{A} = 0$ $\quad \because A.\bar{A} = 0$

$(x+y+z)(x+\bar{y}+\bar{z})(x+y+\bar{z})$

$(x+y+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})$

$(x+y+z.\bar{z})(x+\bar{y}+\bar{z})$ $\quad \because z.\bar{z} = 0$

$(x+y)(x+\bar{y}+\bar{z})$

$x + y(\bar{y}+\bar{z})$

$x + y\bar{y} + y\bar{z}$ $\quad \because y\bar{y} = 0$

$x + y\bar{z}$

⇒ Consensus Theorem

$AB + \bar{A}C + BC = AB + \bar{A}C$

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$(A+B)(\bar{A}+C)(B+C)$

$= (A+B)(\bar{A}+C)$

~~$AB+\bar{A}C$~~

Laws of Consensus Theorem

$AB+\bar{A}C+BC = AB+\bar{A}C$

$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

$(\bar{A}+\bar{B})(B+\bar{C})(\bar{A}+\bar{C})$

$= (\bar{A}+\bar{B})(B+\bar{C})$

$AB+\overline{AB}\,CD$ $\quad \because A+\bar{A}=1$

$= (AB+\overline{AB})(AB+CD)$ $\quad \because AB+\overline{AB}=1$

$= AB+CD$

⇒ Demorgan's Theorem

$\overline{A+B} = \bar{A}\bar{B}$

$\overline{A.B} = \bar{A}+\bar{B}$

$(A+B)(A+B+C)+\bar{A}B$

$(A+B+0)(A+B+C)+\bar{A}B$ $\quad \because 0.C=0$

$(A+B+0.C)+\bar{A}B$

$(A+B)+\bar{A}B$

~~$A\bar{A}B + \bar{A}B.B$~~ $\quad$ ~~$\because A.\bar{A}=0$~~

~~$= \bar{A}B$~~ $\quad$ ~~$B.B=B$~~

$A+B+\bar{A}B$ $\quad \because 1+\bar{A}=1$

$A+B(1+\bar{A}) = A+B$

$A + \bar{A}B + A\bar{B} = A + B$

<u>LHS</u> $A(1+\bar{B}) + \bar{A}B$ $\qquad \because 1+\bar{B} = 1$

$A + \bar{A}B$

$(A+\bar{A})(A+B)$ $\qquad \because A+\bar{A} = 1$

$A+B = \underline{\underline{RHS}}$

$\Rightarrow A + \bar{A}B = A + B$ $\qquad$ Absorption Theorem

$\bar{C} + CB = \bar{C} + B$

$[xy' + xyz + x(y+xy')]'$

$[xy' + xyz + xy + xy']'$ $\qquad \because x.x = x$

$[xy' + xyz + xy]'$ $\qquad \because xy + xy = xy$

$[xy' + xy(z+1)]'$ $\qquad \because z+1 = 1$

$[xy' + xy]'$

$[x(y'+y)]' = x'$ $\qquad \because y'+y = 1$

Minimise the exp and reduce the exp using logic gates

$ABC + AB\bar{C} + A\bar{B}\bar{C}$

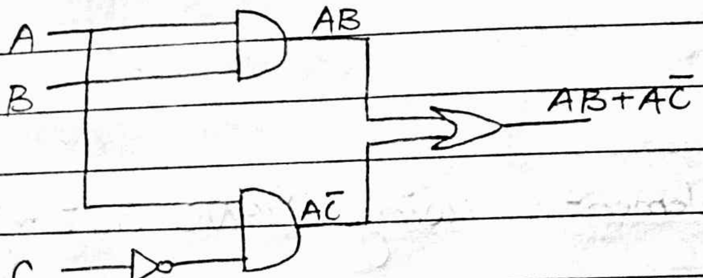
$AB(C+\bar{C}) + A\bar{B}\bar{C}$ $\quad \because C+\bar{C}=1$

$AB + A\bar{B}\bar{C}$

$A(B+\bar{B}\bar{C})$ $\quad \because B+\bar{B}\bar{C}=B+\bar{C}$ [Absorption Th.]

$A(B+\bar{C})$

$AB + A\bar{C}$




$A\bar{B}C + AB\bar{C} + A\bar{B}\bar{C} + AB$

$A\bar{B}(C+\bar{C}) + AB(\bar{C}+1)$ $\quad C+\bar{C}=1$, $\bar{C}+1=1$

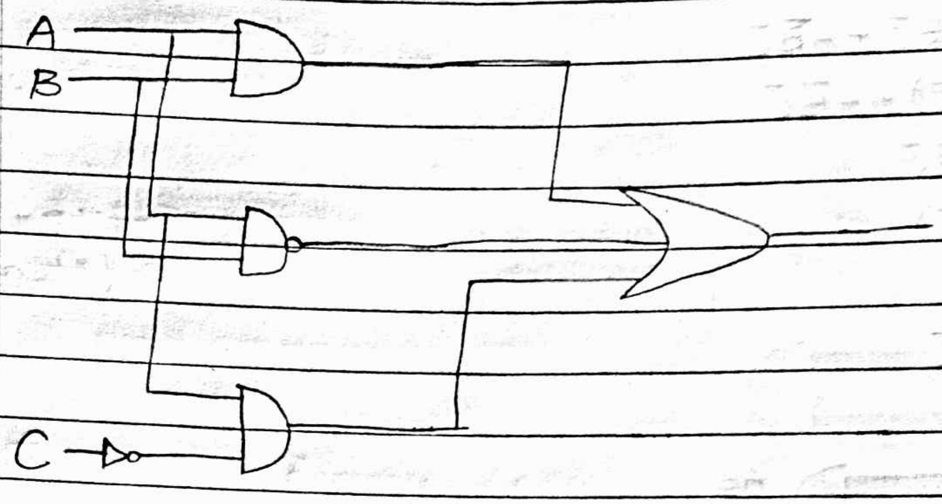
$A\bar{B} + AB$

$A(\bar{B}+B)$ $\quad \bar{B}+B=1$

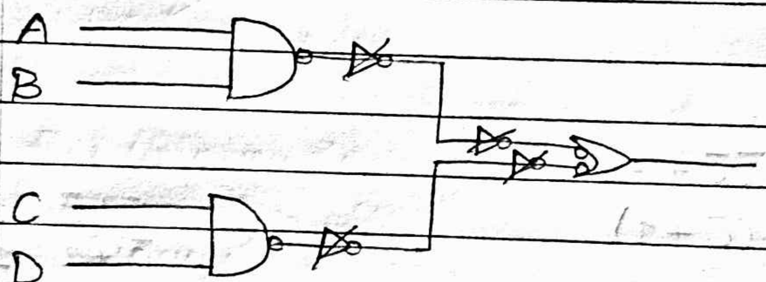
$=A$

A ——— A

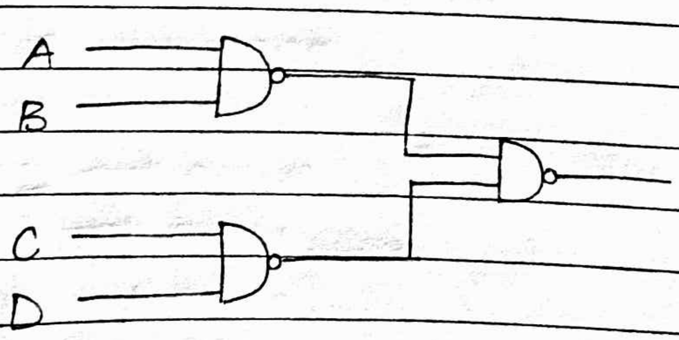
$AB + \overline{AB} + A\overline{C}$




$AB + CD$ Implement using NAND Gate only




Final implementation :-



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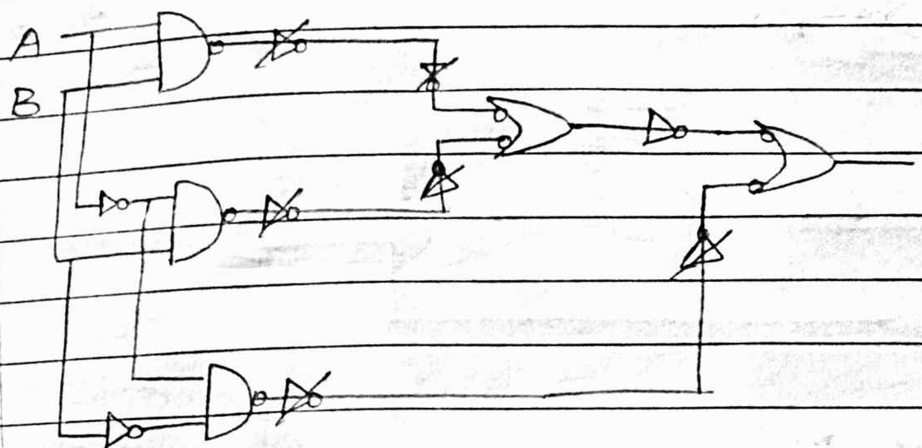
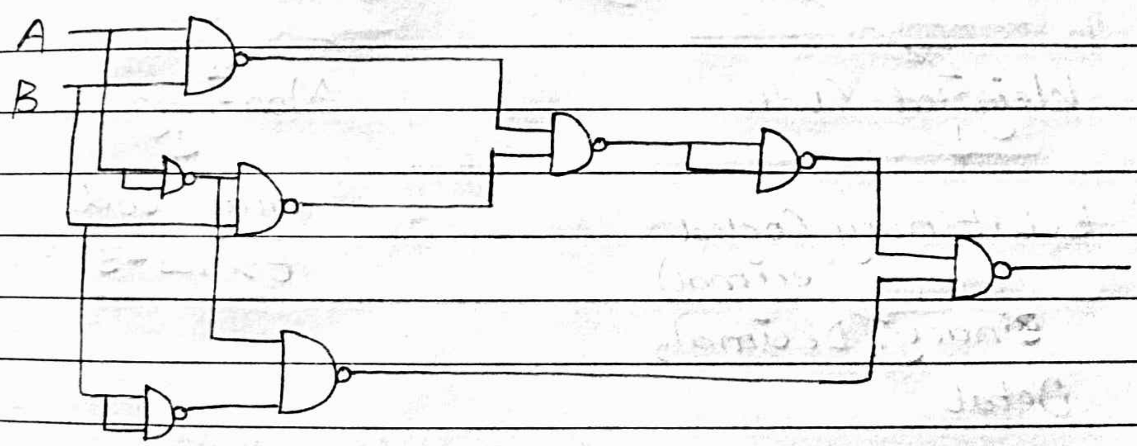
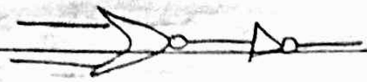
$A B+\bar{A} B+\bar{A} \bar{B}$ Implement only using NAND Gate

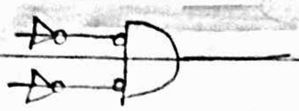



Final implementation



## Using NOR Gate

OR 

AND 

## Types of Binary Codes

Binary Codes

Weighted Codes
- BCD (Binary Coded Decimal)
- Binary, Decimal, Octal

Non-weighted codes
- Gray Code
- Excess-3

Examples:-

| BCD codes | Excess-3 (BCD+0011) |
|---|---|
| 0 → 0000 | 0 → 0011 |
| 1 → 0001 | 1 → 0100 |
| 2 → 0010 | 2 → 0101 |
| 3 → 0011 | 3 → 0110 |
| 4 → 0100 | 4 → 0111 |
| 5 → 0101 | 5 → 1000 |

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Conversion :-

Binary to Gray [use XOR]

11101 ⟶ 10011

1011001 ⟶ 1110101

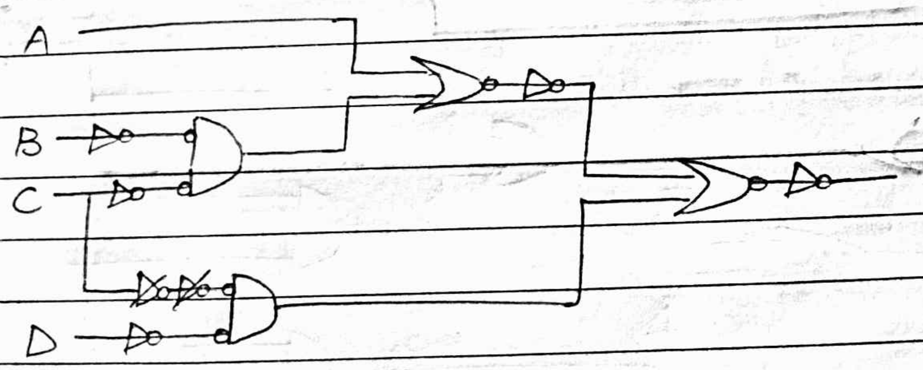
Gray to Binary

1110101

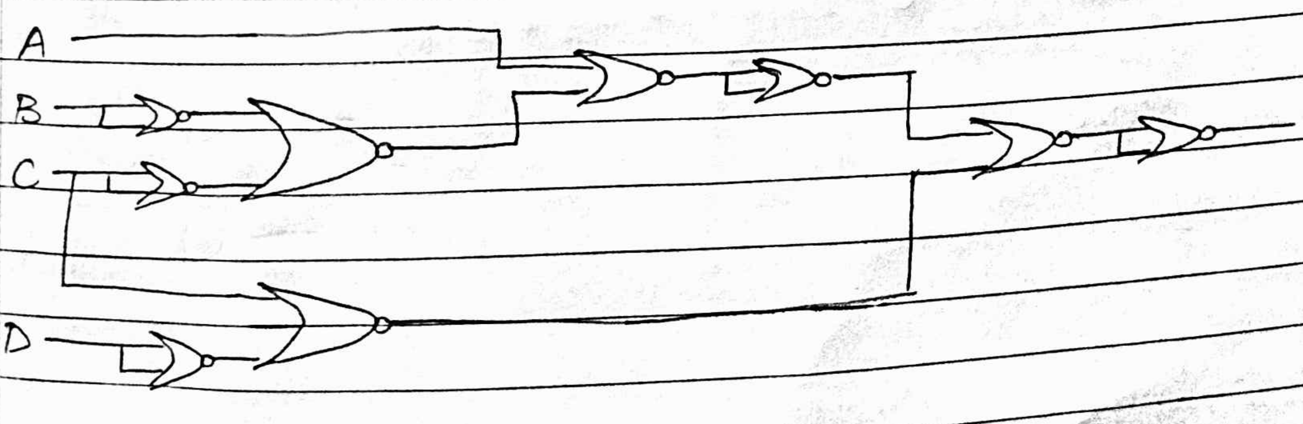
1011001

1110101 ⟶ 1011001

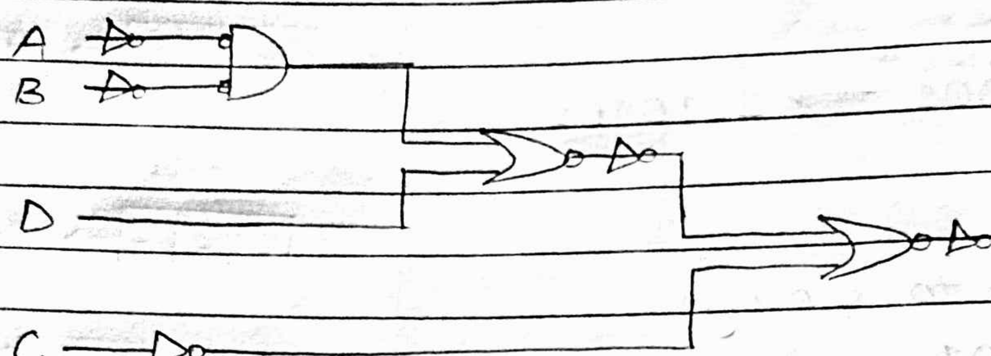
$A+BC+\bar{C}D$ Implement only using NOR Gate



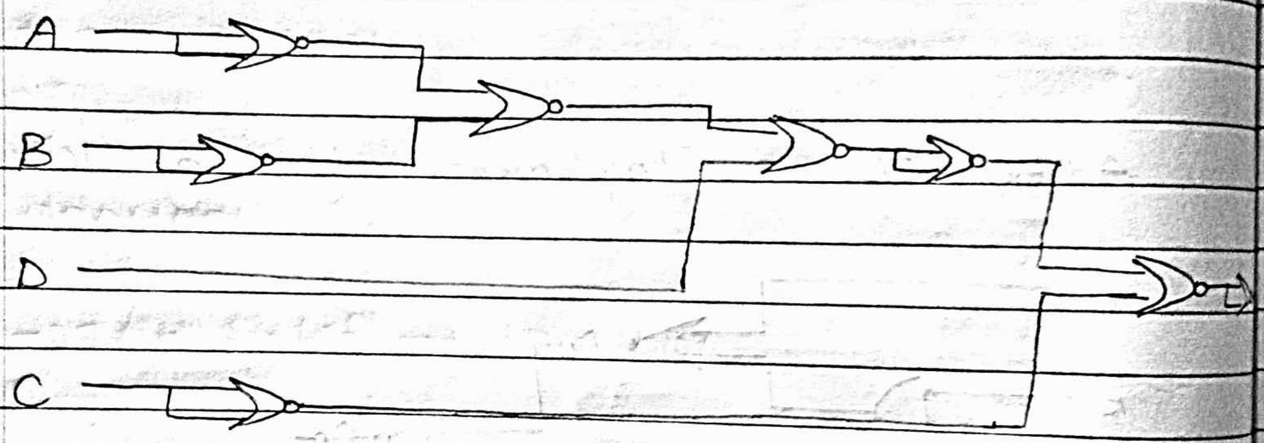

Final implementation :-



$AB + D + \overline{C}$ Implement only using NOR




Final implementation I

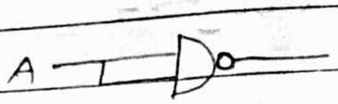



⇒ NAND and NOR are called universal gates because all the other gates can be made using these two gates

# Realisation of basic gates using universal gates

Using NAND | Using NOR

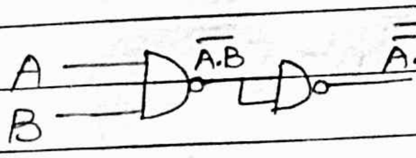
NOT



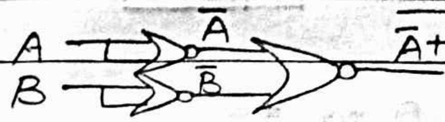

$\overline{A.A} = \overline{A}$


$\overline{A+A} = \overline{A}$

AND

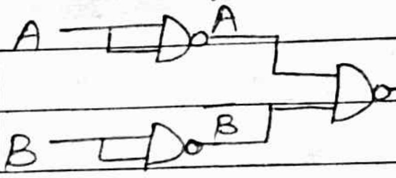



$\overline{\overline{A.B}} = A.B$

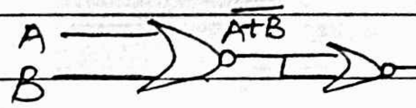



$\overline{\overline{A}+\overline{B}} = A.B$

OR

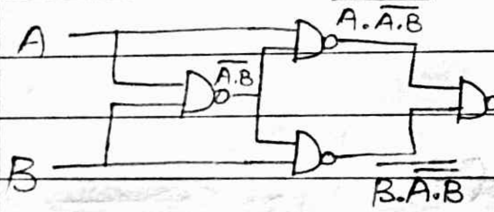


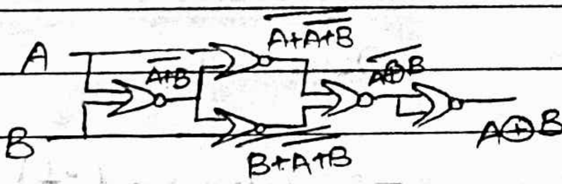

$\overline{\overline{A}.\overline{B}} = A+B$




$\overline{\overline{A+B}} = A+B$

XOR




$\overline{\overline{A.\overline{AB}}.\overline{B.\overline{A.B}}}$

$\overline{\overline{A.\overline{AB}}} + \overline{\overline{B.\overline{A.B}}}$

$A.\overline{AB} + B.\overline{AB} = A(\overline{A}+\overline{B}) + B(\overline{A}+\overline{B})$

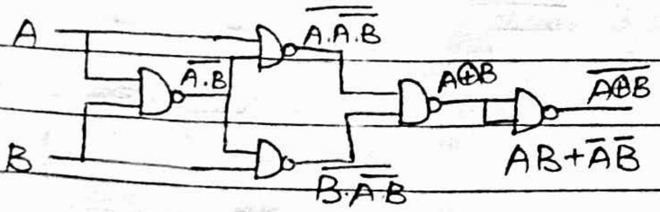
$\cancel{(A+B)(\overline{A}+\overline{B})}$

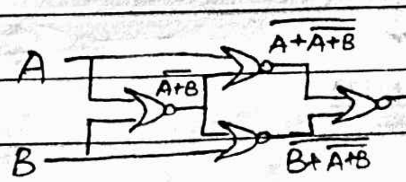
$= A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B}$

$= A\overline{B} + \overline{A}B = A \oplus B$

$\overline{\overline{A \oplus B}} = A \oplus B$

EX-NOR




$(A+\overline{A+B})(B+\overline{A+B})$

$(A+\overline{A}\overline{B})(B+\overline{A}\overline{B}) = AB+\overline{A}\overline{B}$

$= \overline{A \oplus B}$

Numbers

Signed | Unsigned

+4 ← $\boxed{0}\boxed{100}$

MSB for sign | magnitude

$(100)_2 \rightarrow +4$

−4 ← $\boxed{1}\boxed{100}$

0 for +

1 for −

+25 → 011001

↓

in 8 bit

↓

00011001

−25 (in 8-bit) → 10011001

1's complement

−5 → 1101

↓

1's complement

↓

find the complement of +5 → 0101

↓

1010

Note: We always take 1's and 2's complement of a negative number.

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Find 1's complement of
1010 → 0101
001010 → 110101

Find 2's complement of
~~1010 → 10~~
0101 → 1010
+1
1011

Range of 1's complement
$-(2^{n-1}-1)$ to $(2^{n-1}-1)$ n → no. of bits

Find 1's complement of:
-5 -7
+5 = 0101 +7 = 0111
-5 = 1010 -7 = 1000

Solve using 1's complement

Case 1: When carry is generated, add it to LSB

+5−4 = +1 Range → −7 to +7
n = 4
+5 = 0101
+4 = 0100
−4 = 1011

  0101
+ 1011
10000
   +1
0001

[Add the carry to LSB]

Range → -7 to +7
n = 4

+7 − 3 = +4

+7 = 0111

+3 = 0011

−3 = 1100

```
  ¹
  0111
+¹1100
------
  0011
    +1
------
  0100 → +4
```

Range → -31 to 31
n = 6

24 − 13 = +11

+24 = 011000

+13 = 001101

−13 = 110010

```
  ¹
  011000
+¹110010
--------
  001010
      +1
--------
  001011 → +11 Ans
```

Case 2: When no carry is generated and you have 1 on MSB, take 1's complement of the answer and keep the sign same.

−5 + 4 = −1

Range → -7 to 7
n = 4

+5 = 0101

−5 = 1010

+4 = 0100

```
  1010
+ 0100
------
  1110
   ↓
 −001 → −1
```

Range = −15 to 15
n = 5

−9+5 = −4
+9 = 01001
−9 = 10110
+5 = 00101

$$\begin{array}{r} 10110 \\ +\ 00101 \\ \hline 11011 \end{array}$$

↓

10100 → −4 Ans

−44+12 = −32

Range = −63 to 63
n = 7

+44 = 0101100
−44 = 1010011
+12 = 0001100

$$\begin{array}{r} 1010011 \\ +\ 0001100 \\ \hline 1011111 \end{array}$$

↓

1100000 → −32 Ans

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$n=9$

$-128 + 65 = -63$

$+128 = 010000000$

$-128 = 101111111$

$+65 = 001000001$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\,1\,1\,1} \\ 101111111 \\ +\ 001000001 \\ \hline 1\ 11000000 \\ \downarrow \\ 100111111 \end{array}$$

$100111111 \longrightarrow -63$ Ans

Case 3: $-5 - 2 = -7$ $n=4$

combination of case1 and case2

$+5 = 0101$

$-5 = 1010$

$+2 = 0010$

$-2 = 1101$

$$\begin{array}{r} 1010 \\ +\ 1101 \\ \hline 0111 \\ +1 \\ \hline 1000 \\ \downarrow \\ 1111 \end{array}$$

$1111 \rightarrow -7$ Ans

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$n=5$

Case 4: $-5-4=-9$

$+5=00101$

$-5=11010$

$+4=00100$

$-4=11011$

```
   11010
  +11011
   10101
      +1
   10110
     ↓
   11001 → -9 Ans
```

2's compliment

Range $= -(2^{n-1})$ to $(2^{n-1}-1)$

Range $= -8$ to $7$ ($n=4$)

Case 1: $+5-3=+2$

Discard the carry whenever it is generated

$+5=0101$

$+3=0011$

$-3=1100$

```
  1100
    +1
  1101
```

```
   0101
  +1101
   0010 → +2 Ans
```

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Case 2: $+2-6=-4$      $n=4$

Whenever there is 1 on MSB take 2's complement keeping sign same

$+2 = 0010$

$+6 = 0110$

$-6 = 1001$

$\quad\;\; +1$

$\quad 1010$

$\;\;\; 0010$

$+ 1000$

$\;\;\; 1000$

↓

~~XXXX XXXX~~

$1011$

$\;\; +1$

$1100 \rightarrow -4$ Ans

Case 3: $-5-2=-7$      $n=4$

combination of case 1 and 2

$+5 = 0101$

$-5 = 1011$

$+2 = 0010$

$-2 = 1110$

$\;\;\; 1011$

$+ 1110$

$\;\;\; 1001$

↓

$1111 \rightarrow -7$ Ans

$1101$

$\;\; +1$

$1110$

$110$

$\;\;+1$

$111$

$n=5$

Case 4: $-5-4=-9$

$+5=00101$

$-5=11011$

$+4=00100$

$-4=11100$

$$
\begin{array}{r}
\overset{1}{}\;11011 \\
+\;11100 \\
\hline
10111
\end{array}
$$

$\downarrow$

$11001 \rightarrow -9$ Ans

K-map

- → SOP → sum of product
- → POS → product of sum

| SOP | POS |
|---|---|
| Sum of Products | Product of Sum |
| Minterms (m) | Maxterms (M) |
| $\Sigma m$ | $\pi M$ |
| $0 \rightarrow \bar{A}$ | $0 \rightarrow A$ |
| $1 \rightarrow A$ | $1 \rightarrow \bar{A}$ |
| $ABC + A\bar{B}C + \bar{A}\bar{B}C \rightarrow$ SOP | $(A+B+\bar{C})(A+B+C) \leftarrow$ POS |

$0 \to \bar{A}$ $\quad$ $0 \to A$

$1 \to A$ $\quad$ $1 \to \bar{A}$

| xyz | Minterms (SOP) | | Maxterms (POS) | |
|---|---|---|---|---|
| 000 | $\bar{x}\bar{y}\bar{z}$ | $m_0$ | $x+y+z$ | $M_0$ |
| 001 | $\bar{x}\bar{y}z$ | $m_1$ | $x+y+\bar{z}$ | $M_1$ |
| 010 | $\bar{x}y\bar{z}$ | $m_2$ | $x+\bar{y}+z$ | $M_2$ |
| 011 | $\bar{x}yz$ | $m_3$ | $x+\bar{y}+\bar{z}$ | $M_3$ |
| 100 | $x\bar{y}\bar{z}$ | $m_4$ | $\bar{x}+y+z$ | $M_4$ |
| 101 | $x\bar{y}z$ | $m_5$ | $\bar{x}+y+\bar{z}$ | $M_5$ |
| 110 | $xy\bar{z}$ | $m_6$ | $\bar{x}+\bar{y}+z$ | $M_6$ |
| 111 | $xyz$ | $m_7$ | $\bar{x}+\bar{y}+\bar{z}$ | $M_7$ |

Canonical form (Standard form)

$BC + A\bar{B} + AC$

$(A+\bar{A})BC + A\bar{B}(C+\bar{C}) + A(B+\bar{B})C$

$ABC + \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C} + ABC + A\bar{B}C$

$ABC + \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C}$ → canonical form of exp

111 $\quad$ 011 $\quad$ 101 $\quad$ 100

$m_7$ $\quad$ $m_3$ $\quad$ $m_5$ $\quad$ $m_4$

$\Sigma m(3,4,5,7)$ → standard form

$A\bar{B} + B + BC$

$A\bar{B}(C+\bar{C}) + (A+\bar{A})B(C+\bar{C}) + (A+\bar{A})BC$

$A\bar{B}C + A\bar{B}\bar{C} + ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + ABC + \bar{A}BC$

$A\bar{B}C + A\bar{B}\bar{C} + ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$

101 $\quad$ 100 $\quad$ 111 $\quad$ 110 $\quad$ 011 $\quad$ 010

$m_5$ $\quad$ $m_4$ $\quad$ $m_7$ $\quad$ $m_6$ $\quad$ $m_3$ $\quad$ $m_2$

$\Sigma m(2,3,4,5,6,7)$

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$A B \bar{C} + A \bar{B} D + A \bar{B} \bar{D}$

$A B \bar{C}(D+\bar{D}) + A \bar{B}(C+\bar{C}) D + A \bar{B}(C+\bar{C}) \bar{D}$

$A B \bar{C} D + A B \bar{C} \bar{D} + A \bar{B} C D + A \bar{B} \bar{C} D + A \bar{B} C \bar{D} + A \bar{B} \bar{C} \bar{D}$

1101 1100 1011 1001 1010 1000

$m_{13}$ $m_{12}$ $m_{11}$ $m_9$ $m_{10}$ $m_8$

$\sum m(8, 9, 10, 11, 12, 13)$

$(A+B)(A+\bar{C})$

$(A+B+C\bar{C})(A+B\bar{B}+\bar{C})$

$(A+B+C)(A+B+\bar{C})(A+B+\bar{C})(A+\bar{B}+\bar{C})$

$(A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$

000 001 011

$M_0$ $M_1$ $M_3$

$\pi M(0, 1, 3)$

$(A+B)(\bar{B}+C)(\bar{A}+\bar{C})$

$(A+B+C\bar{C})(A\bar{A}+\bar{B}+C)(\bar{A}+B\bar{B}+\bar{C})$

$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

000 001 010 110 101 111

$M_0$ $M_1$ $M_2$ $M_6$ $M_5$ $M_7$

$\pi M(0, 1, 2, 5, 6, 7)$

$ABC + CD + A\bar{B}D$ → find maxterms

$ABC(D+\bar{D}) + (A+\bar{A})(B+\bar{B})CD + A\bar{B}(C+\bar{C})D$

$ABCD + ABC\bar{D} + ABCD + A\bar{B}CD + \bar{A}BCD + \bar{A}\bar{B}CD + A\bar{B}CD + A\bar{B}\bar{C}D$

$ABCD + ABC\bar{D} + A\bar{B}CD + \bar{A}BCD + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D$

1111 1110 1011 0111 0011 0001

$m_{15}$ $m_{4}$ $m_{11}$ $m_{7}$ $m_{3}$ $m_{1}$

$\pi M(0, 2, 4, 5, 6, 8, 9, 10, 12, 13)$

K-map

| 2-variable | 3-variable | 4-variable |
|---|---|---|
| $2^2 = 4$ | $2^3 = 8$ | $2^4 = 16$ |
| ↓ | ↓ | ↓ |
| 4 cells | 8 cells | 16 cells |

2-variable K-map

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 00 [0] | 01 [1] |
| 1 | 10 [2] | 11 [3] |

SOP

| | A \ B | $\bar{B}$ 0 | B 1 |
|---|---|---|---|
| $\bar{A}$ | 0 | 0 | 1 |
| A | 1 | 2 | 3 |

POS

| | A \ B | B 0 | $\bar{B}$ 1 |
|---|---|---|---|
| A | 0 | 0 | 1 |
| $\bar{A}$ | 1 | 2 | 3 |

## 3-variable K-Map

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 (0) | 001 (1) | 011 (3) | 010 (2) |
| 1 | 100 (4) | 101 (5) | 111 (7) | 110 (6) |

00
01 ← 1
11 ← 1
10 ← 2
↓
Gray code pattern
↓
Change of one bit b/w two consecutive numbers

SOP →

| A \ BC | $\bar{B}\bar{C}$ 00 | $\bar{B}C$ 01 | $BC$ 11 | $B\bar{C}$ 10 |
|---|---|---|---|---|
| $\bar{A}$ 0 | 0 | 1 | 3 | 2 |
| $A$ 1 | 4 | 5 | 7 | 6 |

POS →

| A \ BC | $B+C$ 00 | $B+\bar{C}$ 01 | $\bar{B}+\bar{C}$ 11 | $\bar{B}+C$ 10 |
|---|---|---|---|---|
| $A$ 0 | 0 | 1 | 3 | 2 |
| $\bar{A}$ 1 | 4 | 5 | 7 | 6 |

## 4-variable K-map

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0000 (0) | 0001 (1) | 0011 (3) | 0010 (2) |
| 01 | 0100 (4) | 0101 (5) | 0111 (7) | 0110 (6) |
| 11 | 1100 (12) | 1101 (13) | 1111 (15) | 1110 (14) |
| 10 | 1000 (8) | 1001 (9) | 1011 (11) | 1010 (10) |

4-variable

SOP →

| AB \ CD | $\bar{C}\bar{D}$ 00 | $\bar{C}D$ 01 | $CD$ 11 | $C\bar{D}$ 10 |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ 00 | 0 | 1 | 3 | 2 |
| $\bar{A}B$ 01 | 4 | 5 | 7 | 6 |
| $AB$ 11 | 12 | 13 | 15 | 14 |
| $A\bar{B}$ 10 | 8 | 9 | 11 | 10 |

POS →

| AB \ CD | $C+D$ 00 | $C+\bar{D}$ 01 | $\bar{C}+\bar{D}$ 11 | $\bar{C}+D$ 10 |
|---|---|---|---|---|
| $A+B$ 00 | 0 | 1 | 3 | 2 |
| $A+\bar{B}$ 01 | 4 | 5 | 7 | 6 |
| $\bar{A}+\bar{B}$ 11 | 12 | 13 | 15 | 14 |
| $\bar{A}+B$ 10 | 8 | 9 | 11 | 10 |

Note: Minterms are denoted using 1 and maxterms using 0.

Note: Diagonal pairing can't be there in K-map

Reduce the boolean expression using K-map

$\bar{A}B + A\bar{B} + \bar{A}\bar{B}$

01 10 00

$m_1$ $m_2$ $m_0$ $\Sigma m(1,2,0)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $1^{0}$ | $1^{1}$ |
| 1 | $1^{2}$ | $^{3}$ |

$Y = \bar{A} + \bar{B}$

~~$\Sigma m(1,2,3)$~~

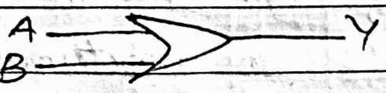
| A \ B | 0 | 1 |
|---|---|---|
| 0 | $1^{0}$ | $^{1}$ |
| 1 | $1^{2}$ | $1^{3}$ |

~~$Y = A + \bar{B}$~~

$\Sigma m(1,2,3)$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $^{0}$ | $1^{1}$ |
| 1 | $1^{2}$ | $1^{3}$ |

$Y = A + B$




| A \ B | | |
|---|---|---|
| | $1^{0}$ | $1^{1}$ |
| | $1^{2}$ | $1^{3}$ |

$Y = 1$

| A \ BC | | | | |
|---|---|---|---|---|
| | $1^{0}$ | $1^{1}$ | $1^{3}$ | $1^{2}$ |
| | $1^{4}$ | $1^{5}$ | $1^{7}$ | $1^{6}$ |

$Y = 1$

**Note:** Priority in pairing is first given to octet, then to quad then to pair

$\bar{A}BC + AB\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

011 110 001 100 111

$m_3$ $m_6$ $m_1$ $m_4$ $m_7$

$\sum m(1, 3, 4, 6, 7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $^{0}$ | $1^{1}$ | $1^{3}$ | $^{2}$ |
| 1 | $1^{4}$ | $^{5}$ | $1^{7}$ | $1^{6}$ |

$Y = \bar{A}C + BC + A\bar{C}$

Redundant pair

In any octet, quad or pair there should be minimum one minterm (1) or maxterm (0) that is not involved in any other pairing.

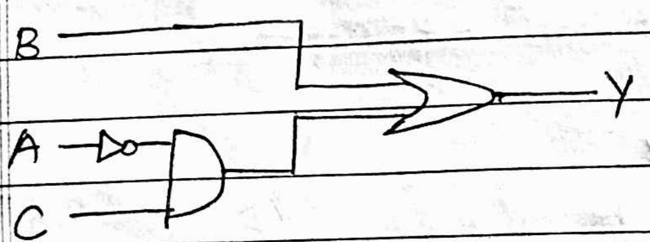
$\sum m(1, 2, 3, 6, 7, 8)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $^{0}$ | $1^{1}$ | $1^{3}$ | $1^{2}$ |
| 01 | $^{4}$ | $^{5}$ | $1^{7}$ | $1^{6}$ |
| 11 | $^{12}$ | $^{13}$ | $^{15}$ | $^{14}$ |
| 10 | $1^{8}$ | $^{9}$ | $^{11}$ | $^{10}$ |

$\Sigma m(1, 2, 3, 6, 7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | 1 | 1 |
| 1 | | | 1 | 1 |

$Y = B + \bar{A}C$



$A\bar{B}C + \bar{A}BC + ABC + AB\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C}$

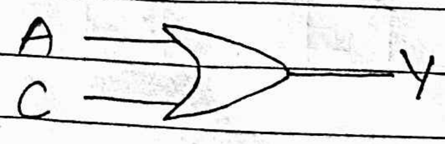
101   011   111   ~~001~~ 110   001   100

$m_5$   $m_3$   $m_7$   $m_6$   $m_1$   $m_4$

$\Sigma m(1, 3, 4, 5, 6, 7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | 1 | |
| 1 | 1 | 1 | 1 | 1 |

$Y = A + C$



$(A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(A+B+C)(\bar{A}+\bar{B}+C)(A+\bar{B}+C)$

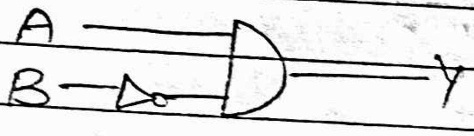
001 011 111 000 110 010

$M_1$ $M_3$ $M_7$ $M_0$ $M_6$ $M_2$

$\pi M(0,1,2,3,6,7)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 (0) | 0 (1) | 0 (3) | 0 (2) |
| 1 | (4) | (5) | 0 (7) | 0 (6) |

$Y = A\bar{B}$




$\Sigma m(1,2,3,5,6,7,9,10,12,12,15)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | (0) | 1 (1) | 1 (3) | 1 (2) |
| 01 | (4) | 1 (5) | 1 (7) | 1 (6) |
| 11 | 1 (12) | (13) | 1 (15) | (14) |
| 10 | (8) | 1 (9) | 1 (11) | 1 (10) |

$Y = CD + \bar{A}C + \bar{A}D + \bar{B}D + \bar{B}C + AB\bar{C}\bar{D}$

$\pi M(1,3,5,6,9,10)$ Reduce exp using K-map and find the ans in SOP

$\Sigma m(0,2,4,7,8,11,12,13,14,15)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 (0) | (1) | (3) | 1 (2) |
| 01 | 1 (4) | (5) | 1 (7) | (6) |
| 11 | 1 (12) | 1 (13) | 1 (15) | 1 (14) |
| 10 | 1 (8) | (9) | 1 (11) | (10) |

$$Y = \bar{C}\bar{D} + AB + ACD + BCD + \bar{A}\bar{B}\bar{D}$$

$\Sigma m(0,1,2,3,5,6,9)$ Reduce the exp using K-map and find the ans in POS

$\pi M(4,7,8,10,11,12,13,14,15)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | (0) | (1) | (3) | (2) |
| 01 | 0 (4) | (5) | 0 (7) | (6) |
| 11 | 0 (12) | 0 (13) | 0 (15) | 0 (14) |
| 10 | 0 (8) | (9) | 0 (11) | 0 (10) |

$$Y = (\overline{A}+\overline{B})(\bar{A}+\bar{C})(\bar{B}+\bar{C}+\bar{D})(\bar{A}+D)(\bar{B}+C+D)$$

$\Sigma m(0, 1, 2, 3, 5, 9, 10, 12, 14, 15)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 (0) | 1 (1) | 1 (3) | 1 (2) |
| 01 | (4) | 1 (5) | (7) | (6) |
| 11 | 1 (12) | (13) | 1 (15) | 1 (14) |
| 10 | (8) | 1 (9) | (11) | 1 (10) |

$$Y = \bar{A}\bar{B} + \bar{A}\bar{C}D + \bar{B}\bar{C}D + AB\bar{D} + ABC + AC\bar{D}$$

$\Sigma m(1, 3, 5, 7, 13, 15, 9, 11) + d(4, 6)$
, 14

don't care terms

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | (0) | 1 (1) | 1 (3) | (2) |
| 01 | X (4) | 1 (5) | 1 (7) | X (6) |
| 11 | (12) | 1 (13) | 1 (15) | 1 (14) |
| 10 | (8) | 1 (9) | 1 (11) | (10) |

$$Y = D + BC$$

**Note:** No two or more than two don't care terms can pair ~~with~~ among themselves.

$\Sigma m(1, 3, 5, 6, 10, 11, 12, 13, 14) + d(7, 0)$

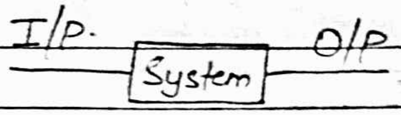
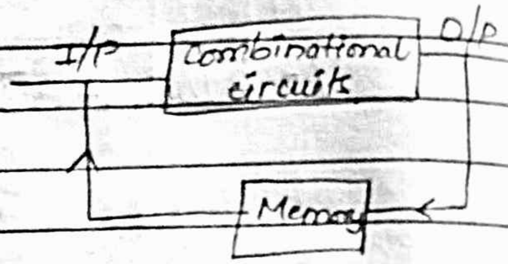
| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | X $^{0}$ | 1 $^{1}$ | 1 $^{3}$ | $^{2}$ |
| 01 | $^{4}$ | 1 $^{5}$ | X $^{7}$ | 1 $^{6}$ |
| 11 | 1 $^{12}$ | 1 $^{13}$ | $^{15}$ | 1 $^{14}$ |
| 10 | $^{8}$ | $^{9}$ | 1 $^{11}$ | 1 $^{10}$ |

→ Reduntant pair

$Y = \bar{A}D + AB\bar{C} + BC\bar{D} + A\bar{B}C$

# UNIT-2

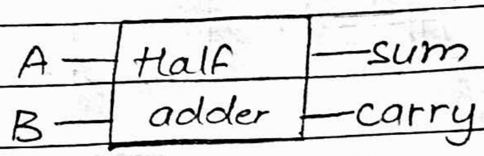
Digital Circuits
- Combinational
- Sequential

| Combinational | Sequential |
| --- | --- |
|   |   |
| → Here, output depends only on present input. | |
| → No memory element | Output depends on the present input as well as the past outputs. |
| → No feedback | Feedback given through the memory element. |
| → Time independent | Time dependent |
| → Fast | Slow |
| → Not complex | Complex |

**Note:** Basic building block of logic gate → Flip flop

Half adder [2-bit addition]

I. Block diagram




II Truth Table

| A | B | sum | carry |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

III Expression for sum and carry

Exp for sum

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $^{0}$ | 1 $^{1}$ |
| 1 | 1 $^{2}$ | $^{3}$ |

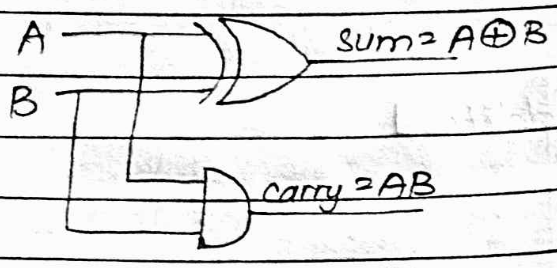
$sum = \overline{A}B + A\overline{B} = A \oplus B$

Exp for carry

| A \ B | 0 | 1 |
|---|---|---|
| 0 | $^{0}$ | $^{1}$ |
| 1 | $^{2}$ | 1 $^{3}$ |

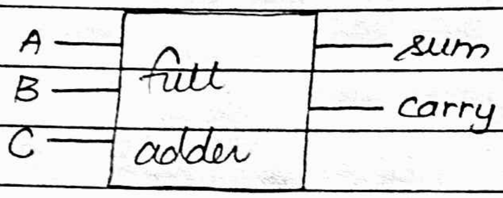
$carry = AB$

IV Implementation of half adder




Full Adder

I. Block diagram




II Truth Table

| A B C | sum | carry |
|---|---|---|
| 0 0 0 | 0 | 0 |
| 0 0 1 | 1 | 0 |
| 0 1 0 | 1 | 0 |
| 0 1 1 | 0 | 1 |
| 1 0 0 | 1 | 0 |
| 1 0 1 | 0 | 1 |
| 1 1 0 | 0 | 1 |
| 1 1 1 | 1 | 1 |

$sum = \Sigma m(1,2,4,7)$

$carry = \Sigma m(3,5,6,7)$

## III Exp for sum

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | (0) | 1 (1) | (3) | 1 (2) |
| 1 | 1 (4) | (5) | 1 (7) | (6) |

$$\text{sum} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC = \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$$

$$\bar{A}(B \oplus C) + A(\overline{B \oplus C})$$

Let $B \oplus C = x$

$$\bar{A}x + A\bar{x} = A \oplus x$$

$$= A \oplus B \oplus C$$

Exp for carry

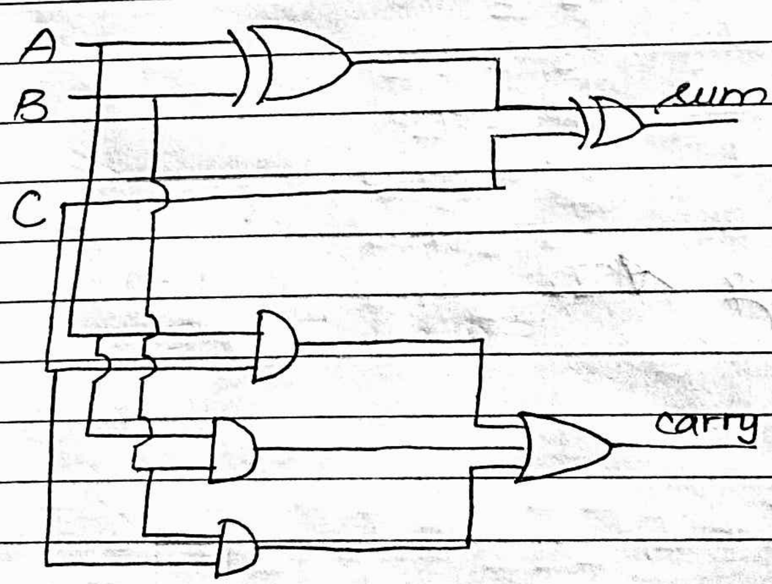
| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | (0) | (1) | 1 (3) | (2) |
| 1 | (4) | 1 (5) | 1 (7) | 1 (6) |

~~carry = $\bar{A}\bar{B}\bar{C}$ +~~

$$\text{carry} = AC + AB + BC$$

## IV Implementing using AOI

$$\text{sum} = A \oplus B \oplus C$$



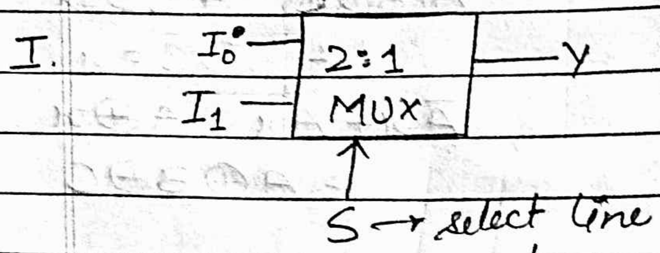
# Combinational Circuits

- MUX
- DEMUX
- ENCODER
- DECODER

## Multiplexer → MUX → Many to One

2:1 MUX

I.




2:1 → outputs (1 → no. of select lines; 2 → input)

decide which data will be rooted as output

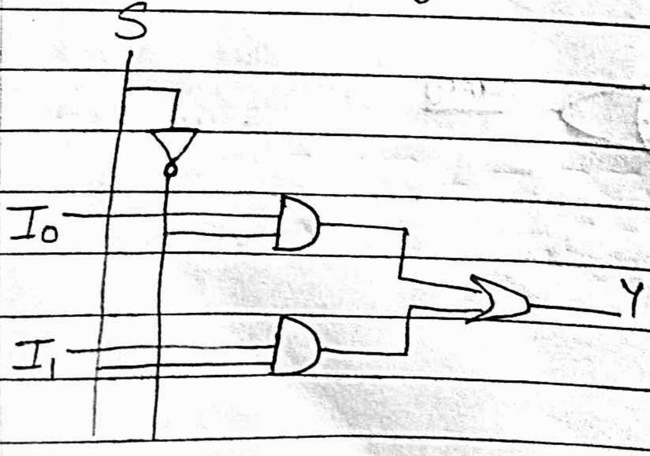
II. Truth Table

| I | S | Y |
|---|---|---|
| $I_0$ | 0 | $I_0\bar{S}$ |
| $I_1$ | 1 | $I_1 S$ |

III Expression

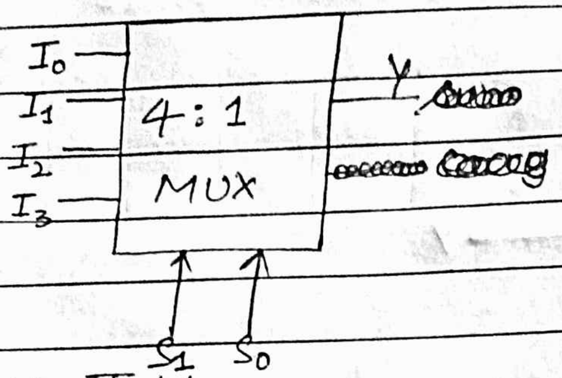
$$Y = I_0\bar{S} + I_1 S$$

IV. Implementing using AOT



## 4:1 MUX

I.



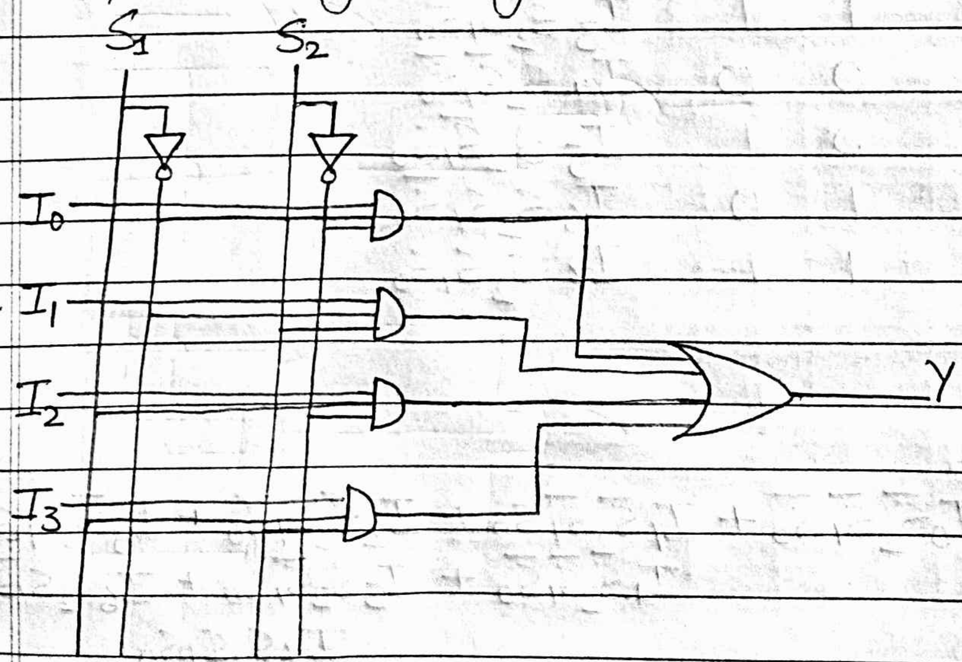

II. Truth Table

| I | $S_1$ | $S_0$ | Y |
|---|---|---|---|
| $I_0$ | 0 | 0 | $I_0\overline{S_1}\,\overline{S_0}$ |
| $I_1$ | 0 | 1 | $I_1\overline{S_1}S_0$ |
| $I_2$ | 1 | 0 | $I_2S_1\overline{S_0}$ |
| $I_3$ | 1 | 1 | $I_3S_1S_0$ |

III. Expression

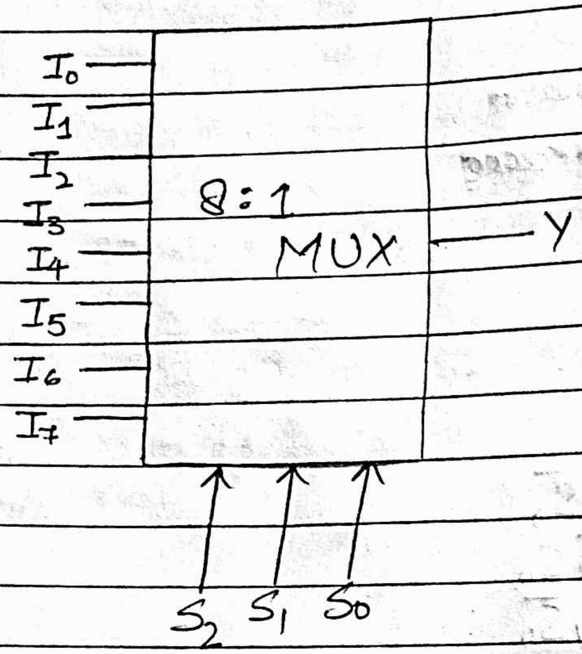
$$Y = I_0\overline{S_1}\,\overline{S_0} + I_1\overline{S_1}S_0 + I_2S_1\overline{S_0} + I_3S_1S_0$$

IV. Implementing using AOI



# 8:1 MUX

I.



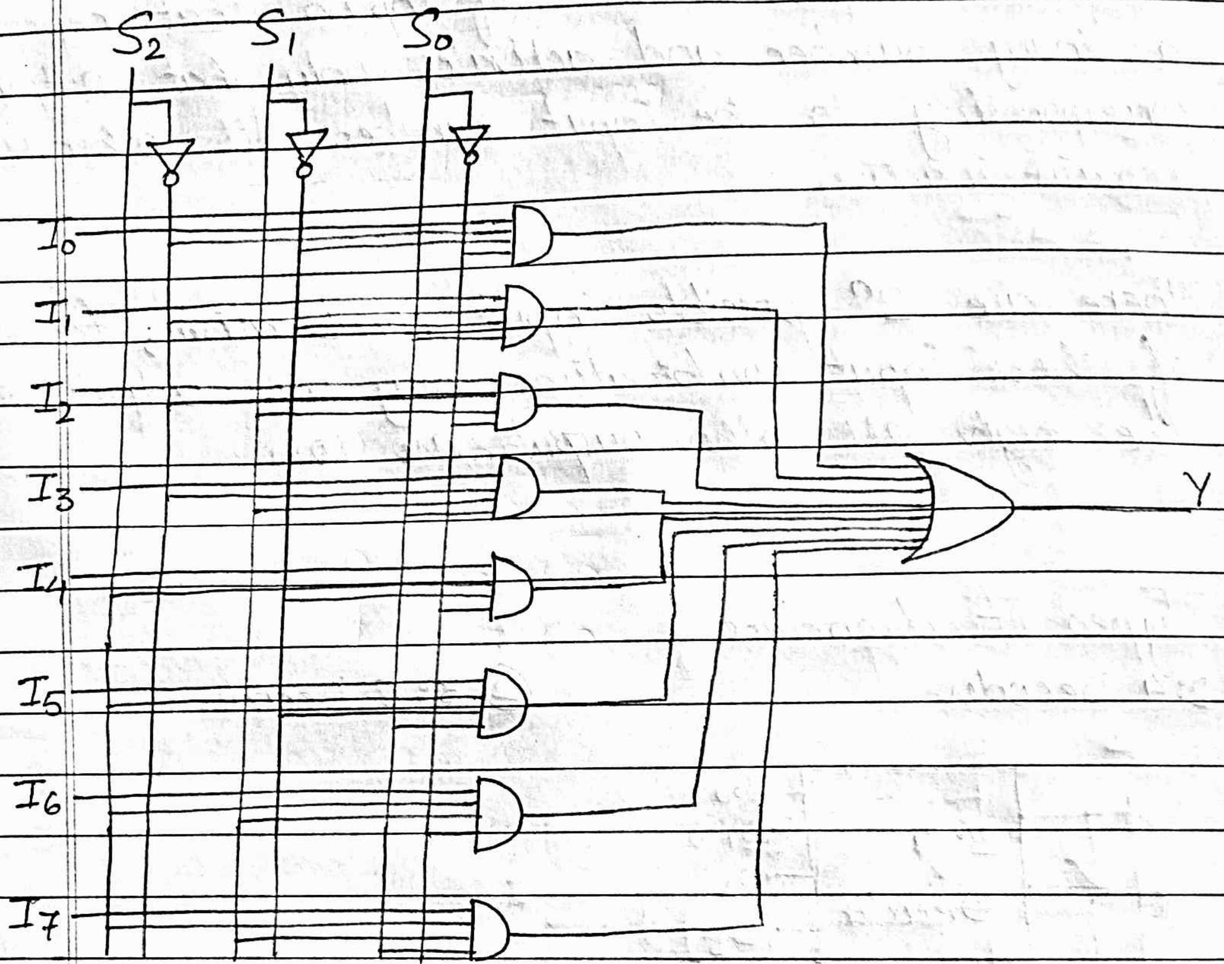

II Truth Table

| I | $S_2$ | $S_2$ | $S_0$ | Y |
|---|---|---|---|---|
| $I_0$ | 0 | 0 | 0 | $I_0\bar{S}_2\bar{S}_1\bar{S}_0$ |
| $I_1$ | 0 | 0 | 1 | $I_1\bar{S}_2\bar{S}_1S_0$ |
| $I_2$ | 0 | 1 | 0 | $I_2\bar{S}_2S_1\bar{S}_0$ |
| $I_3$ | 0 | 1 | 1 | $I_3\bar{S}_2S_1S_0$ |
| $I_4$ | 1 | 0 | 0 | $I_4S_2\bar{S}_1\bar{S}_0$ |
| $I_5$ | 1 | 0 | 1 | $I_5S_2\bar{S}_1S_0$ |
| $I_6$ | 1 | 1 | 0 | $I_6S_2S_1\bar{S}_0$ |
| $I_7$ | 1 | 1 | 1 | $I_7S_2S_1S_0$ |

III Expression

$$Y = I_0\bar{S}_2\bar{S}_1\bar{S}_0 + I_1\bar{S}_2\bar{S}_1S_0 + I_2\bar{S}_2S_1\bar{S}_0 + I_3\bar{S}_2S_1S_0 + I_4S_2\bar{S}_1\bar{S}_0 + I_5S_2\bar{S}_1S_0 + I_6S_2S_1\bar{S}_0 + I_7S_2S_1S_0$$

IV. Implementing using AOI



## Decoder

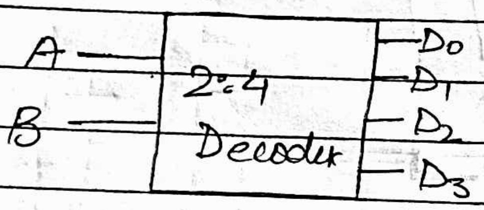
A decoder is a combinational circuit.
A decoder accepts a set of inputs that represents a binary number and activates only that output corresponding to the input number. All other outputs remain inactive.

There are $2^n$ possible input combinations, for each of these input combinations only one output will be high all other outputs are low.

### Types of decoders

① 2:4 Decoder ② 3:8 Decoder

I.




II.

| A | B | $D_0$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |

III. Expression

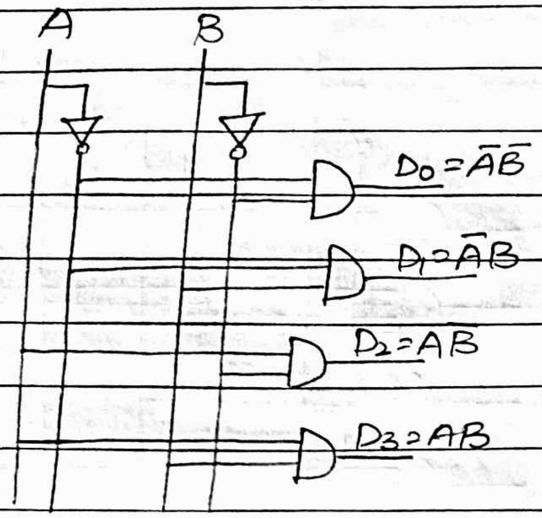
$D_0 = \overline{A}\overline{B}$

$D_1 = \overline{A}B$

$D_2 = A\overline{B}$

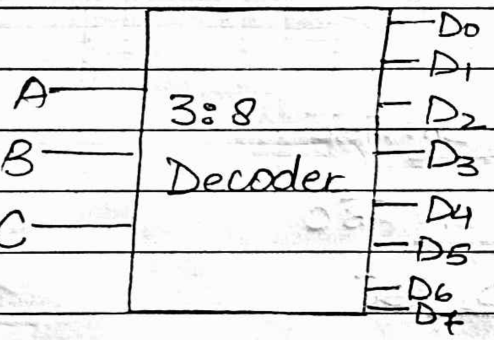
$D_3 = AB$

## IV Implementing using AOI




## 3:8 Decoder

I




II. Truth Table

| A | B | C | $D_0$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $D_6$ | $D_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

## III. Expression

$D_0 = \bar{A}\bar{B}\bar{C}$

$D_1 = \bar{A}\bar{B}C$

$D_2 = \bar{A}B\bar{C}$

$D_3 = \bar{A}BC$

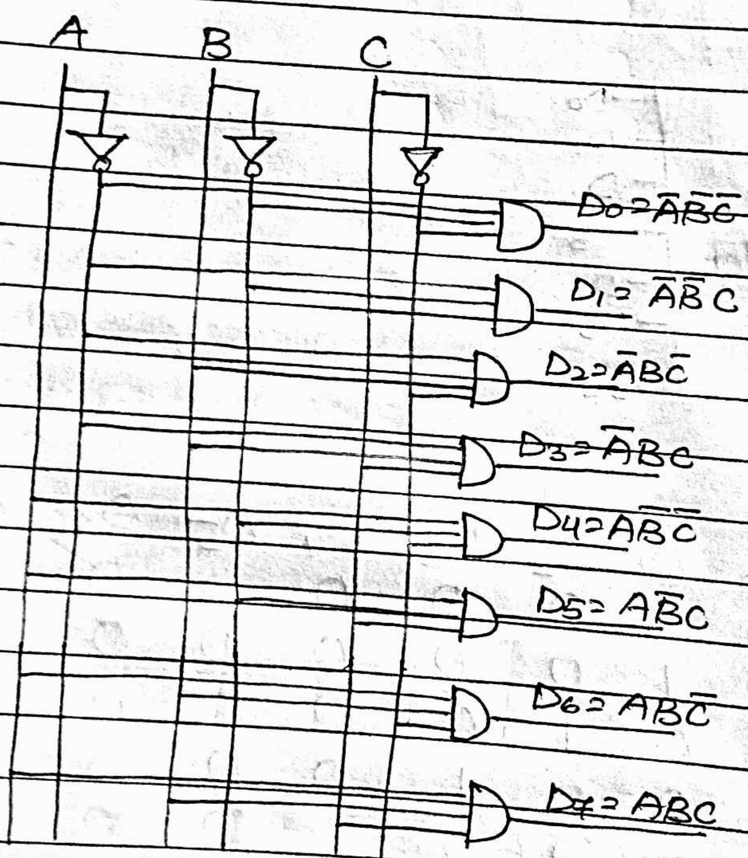
$D_4 = A\bar{B}\bar{C}$

$D_5 = A\bar{B}C$

$D_6 = AB\bar{C}$

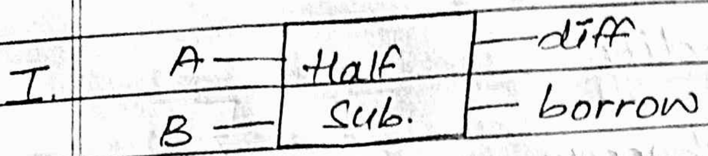
$D_7 = ABC$

## IV. Implementing using AOI



# Subtractor
## Half subtractor

I.




## II Truth Table

| A | B | diff | borrow |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

## III Expression

Exp for diff

| A \ B | 0 | 1 |
|---|---|---|
| 0 | (0) | 1 (1) |
| 1 | 1 (2) | (3) |

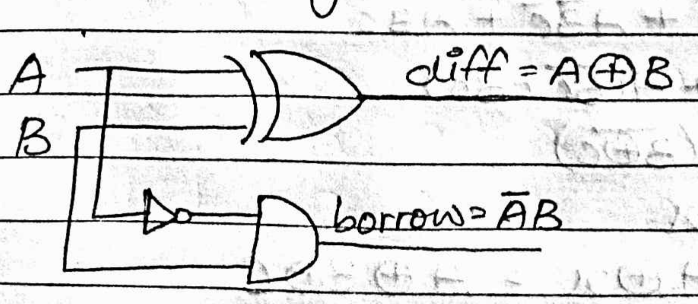
$diff = \bar{A}B + A\bar{B} = A \oplus B$

Exp for borrow

| A \ B | 0 | 1 |
|---|---|---|
| 0 | (0) | 1 (1) |
| 1 | (2) | (3) |

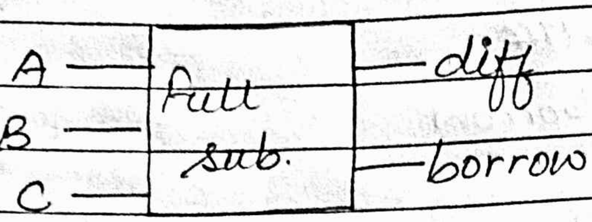
$borrow = \bar{A}B$

## IV Implementing.



# Full Subtractor

## I. Block diagram




## II. Truth Table

| A | B | C | diff | borrow |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

## III Expression

diff $= \Sigma m(1, 2, 4, 7)$

carry $= \Sigma m(1, 2, 3, 7)$

Exp for diff

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | (0) | 1 (1) | (3) | 1 (2) |
| 1 | 1 (4) | (5) | 1 (7) | (6) |

$$\text{diff} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$
$$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$$
$$= \bar{A}(B \oplus C) + A(\overline{B \oplus C})$$

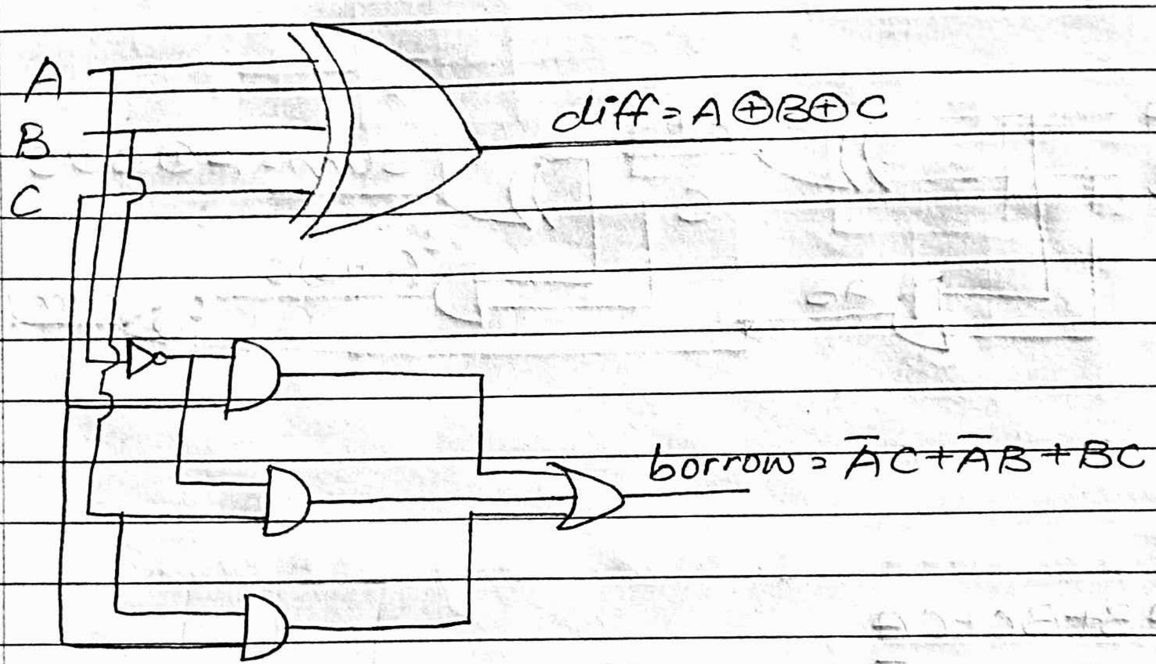
let $B \oplus C = x$

$$\bar{A}x + A\bar{x} = A \oplus x = A \oplus B \oplus C$$

Exp for borrow

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | [0] | 1 [1] | 1 [3] | 1 [2] |
| 1 | [4] | [5] | 1 [7] | [6] |

$borrow = \bar{A}C + \bar{A}B + BC$



## Implement full adder using half adder

HA

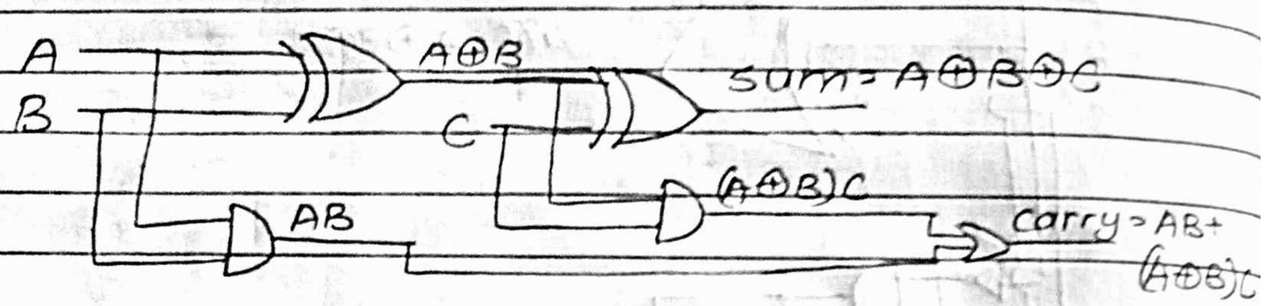
Sum $= A \oplus B$

carry $= AB$

FA

sum $= A \oplus B \oplus C$

carry $= AB + BC + CA$




$AB + BC + CA$

$= ABC + AB\bar{C} + ABC + \bar{A}BC + ABC + A\bar{B}C$

$= ABC + AB\bar{C} + \bar{A}BC + A\bar{B}C$

$= AB(C + \bar{C}) + C(\bar{A}B + A\bar{B})$

$= AB + C(A \oplus B)$

# Implement full subtractor using half subtractor

HS

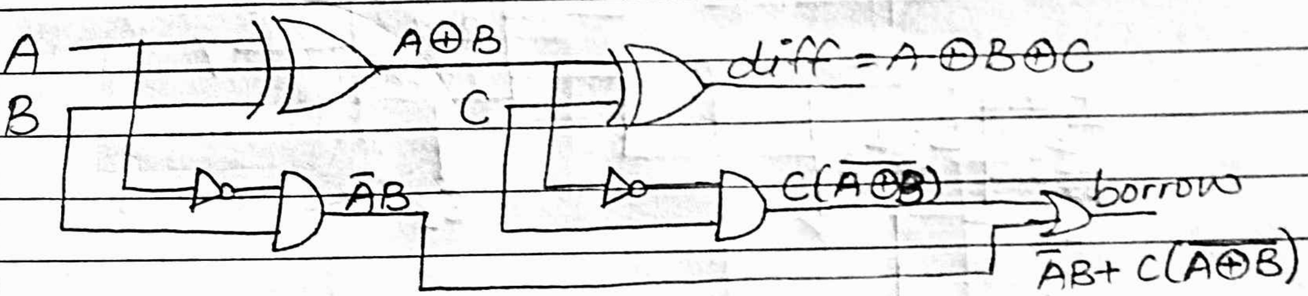
diff = $A \oplus B$

borrow = $\bar{A}B$

FS

diff = $A \oplus B \oplus C$

borrow = $\bar{A}C + \bar{A}B + BC$




$\bar{A}C + \bar{A}B + BC$

$\bar{A}BC + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + \bar{A}BC$

$\bar{A}BC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC + \phi$

$\bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

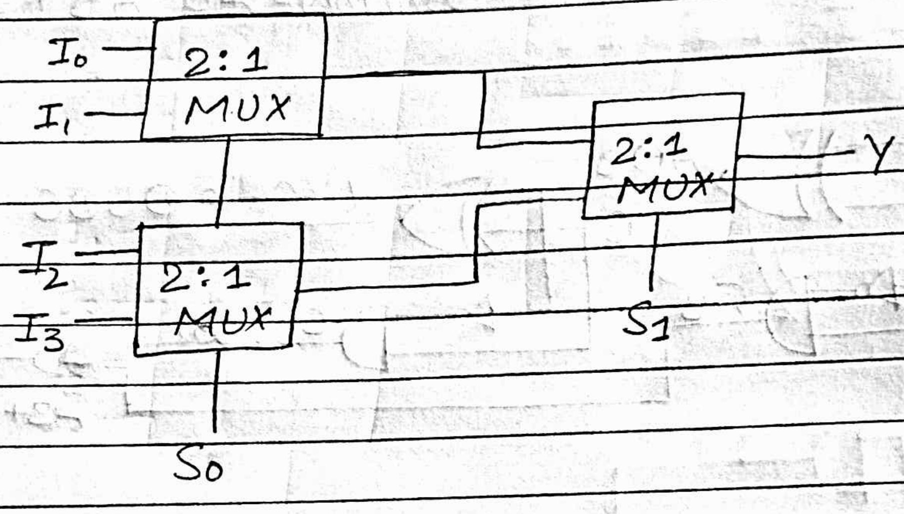
$\bar{A}(BC + B\bar{C}) + C(\bar{A}\bar{B} + AB)$

$\bar{A}B(C + \bar{C}) + C(\overline{A \oplus B})$

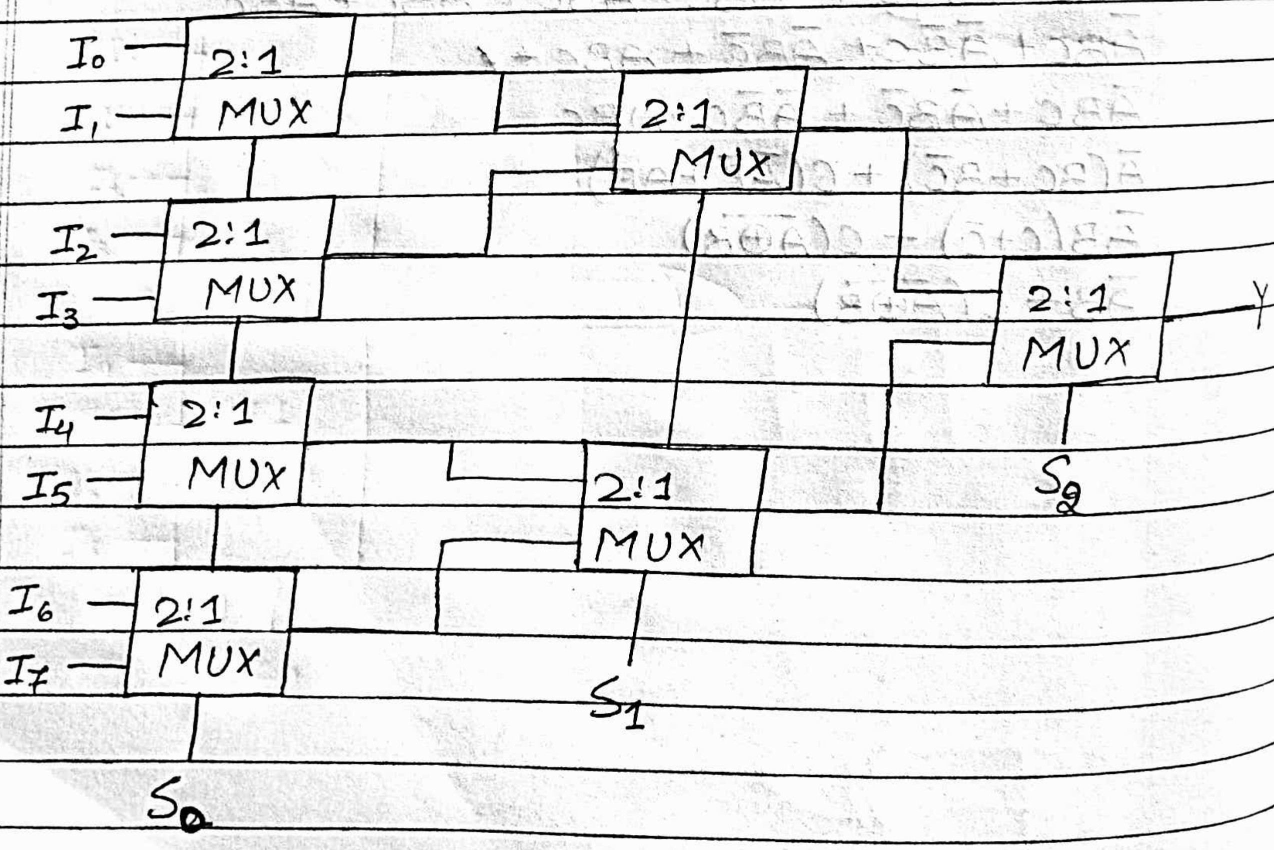
$\bar{A}B + C(\overline{A \oplus B})$

## MUX Tree

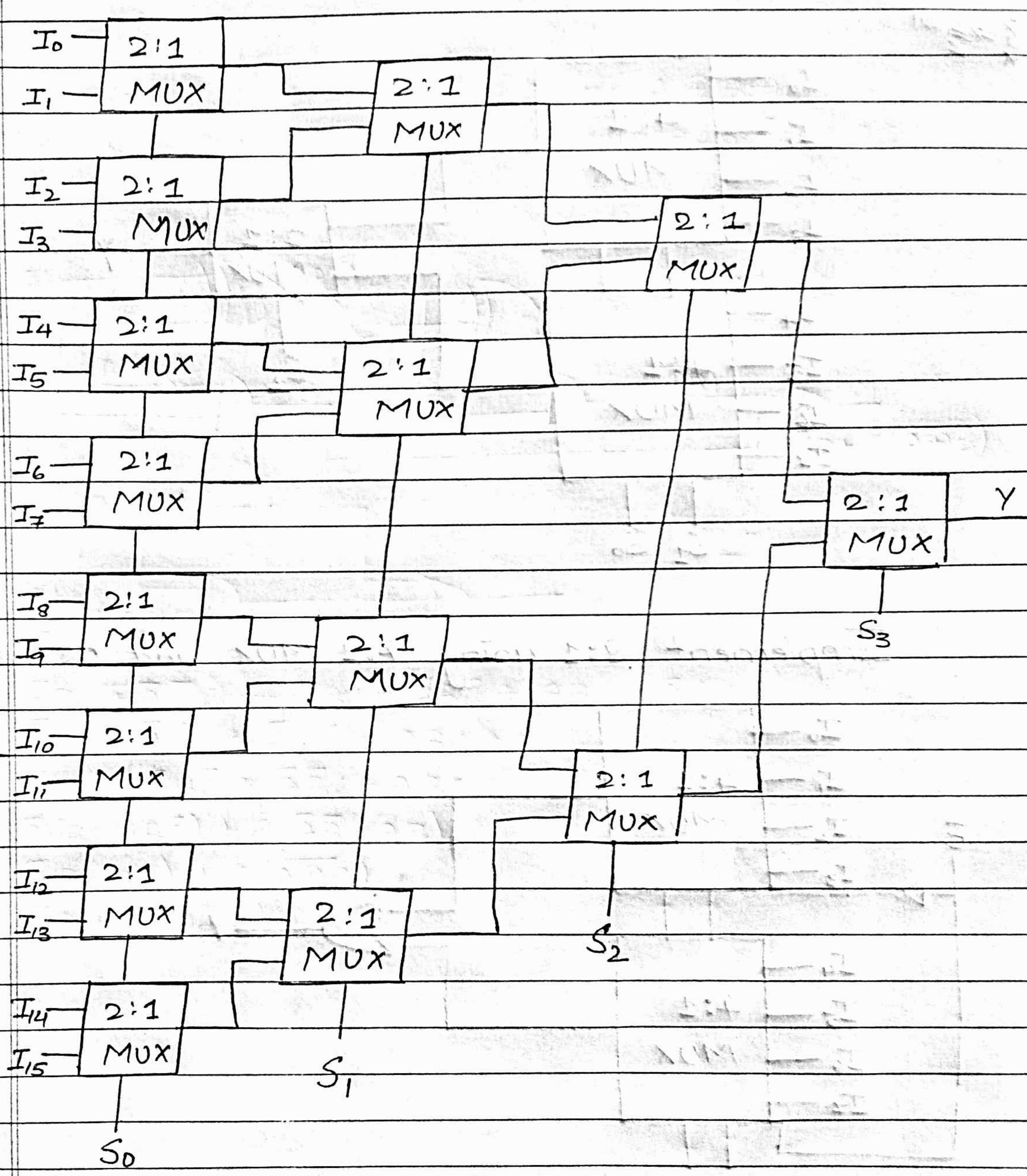
Implement 4:1 MUX using 2:1 MUX



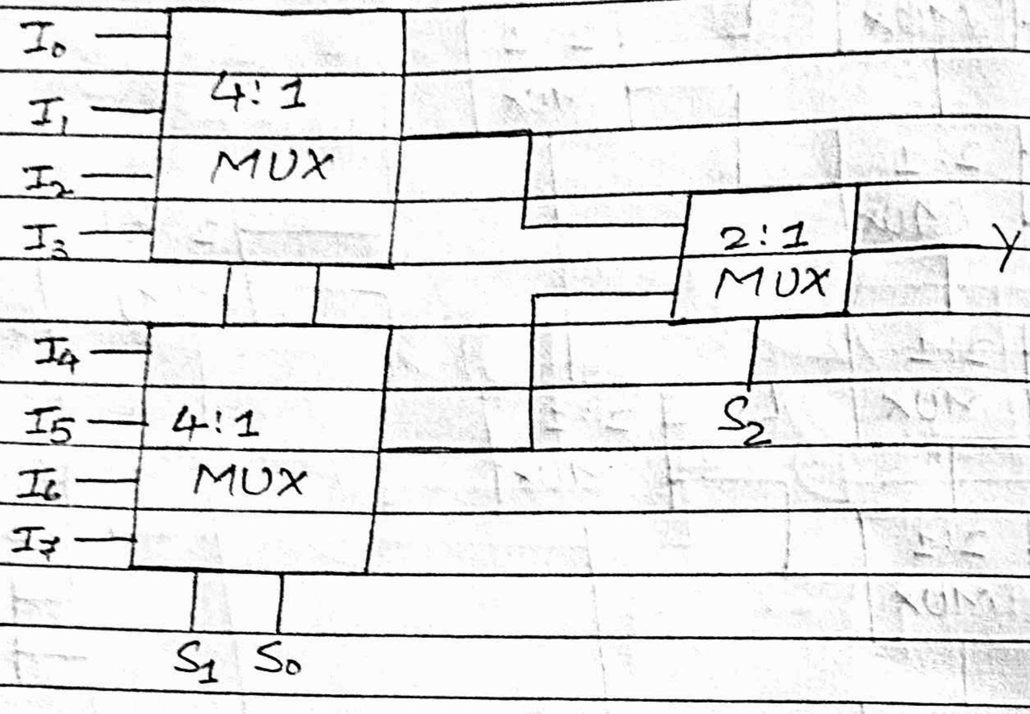

Implement 8:1 MUX using 2:1 MUX



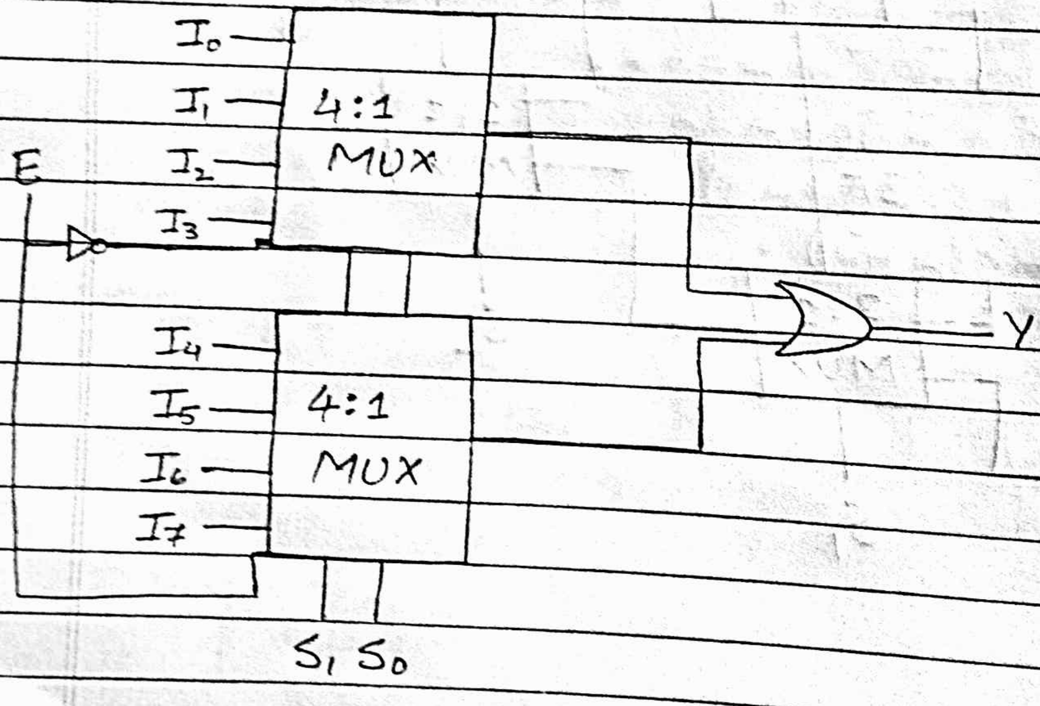
Implement 16:1 MUX using 2:1 MUX



Implement 8:1 using 4:1 MUX




Implement 8:1 using 4:1 MUX and OR Gate only



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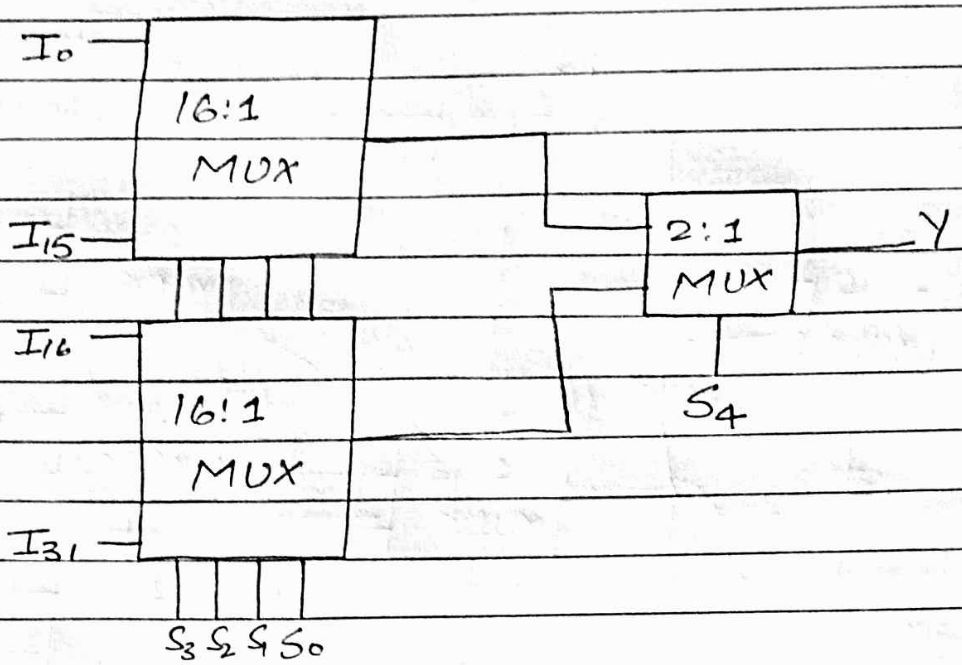
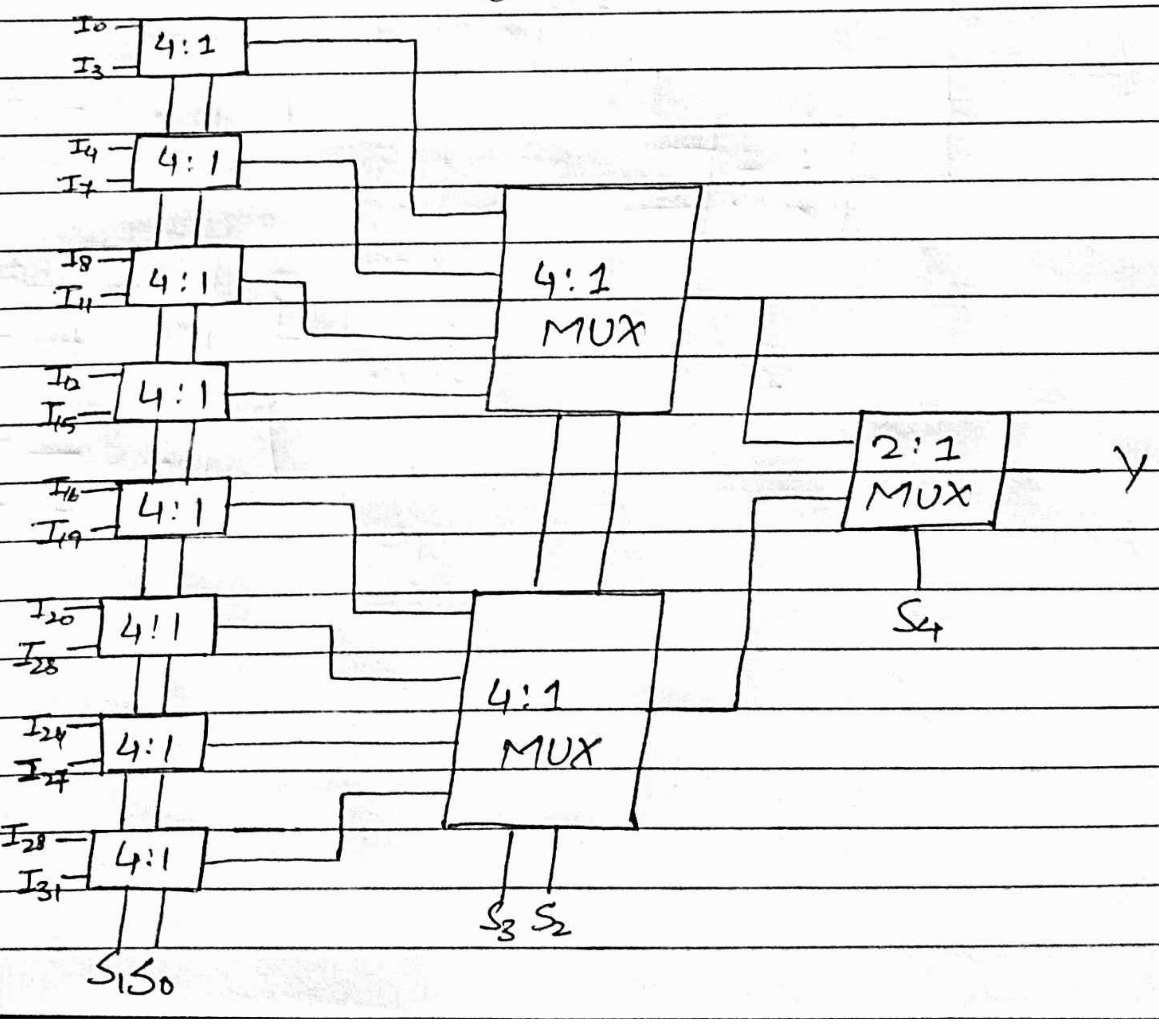
Implement 32:1 using 16:1 MUX




Implement 32:1 using 4:1 MUX



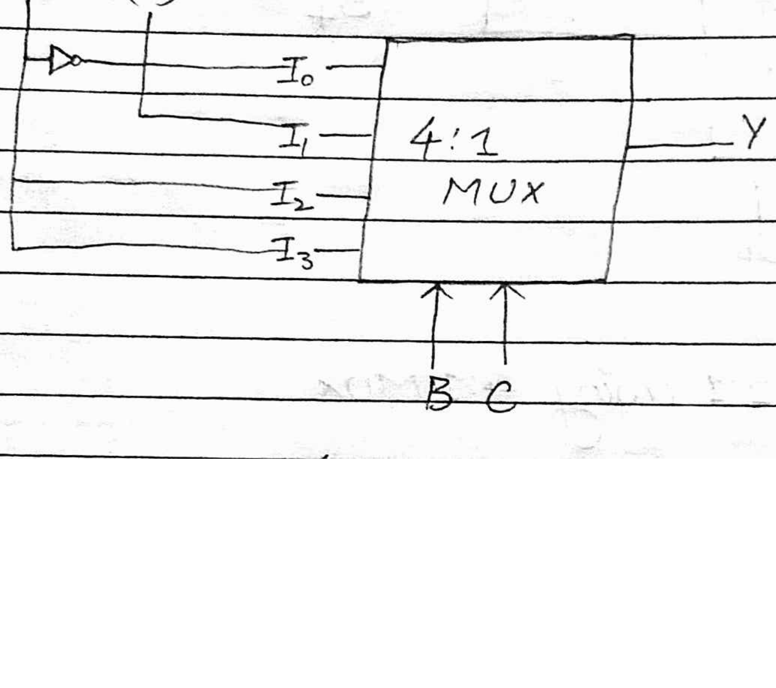
$f(A, B, C) = \sum m(0, 1, 5, 6, 7)$ Implement using 4:1

| A \ BC | $I_0$ 00 | $I_1$ 01 | $I_2$ 10 | $I_3$ 11 |
|---|---|---|---|---|
| 0 | 1 $^{0}$ | 1 $^{1}$ | $^{2}$ | $^{3}$ |
| 1 | $^{4}$ | 1 $^{5}$ | 1 $^{6}$ | 1 $^{7}$ |
| | $\bar{A}$ | $A+\bar{A}=1$ | $A$ | $A$ |

4:1
↓
2 select lines

BC → select lines

A → input

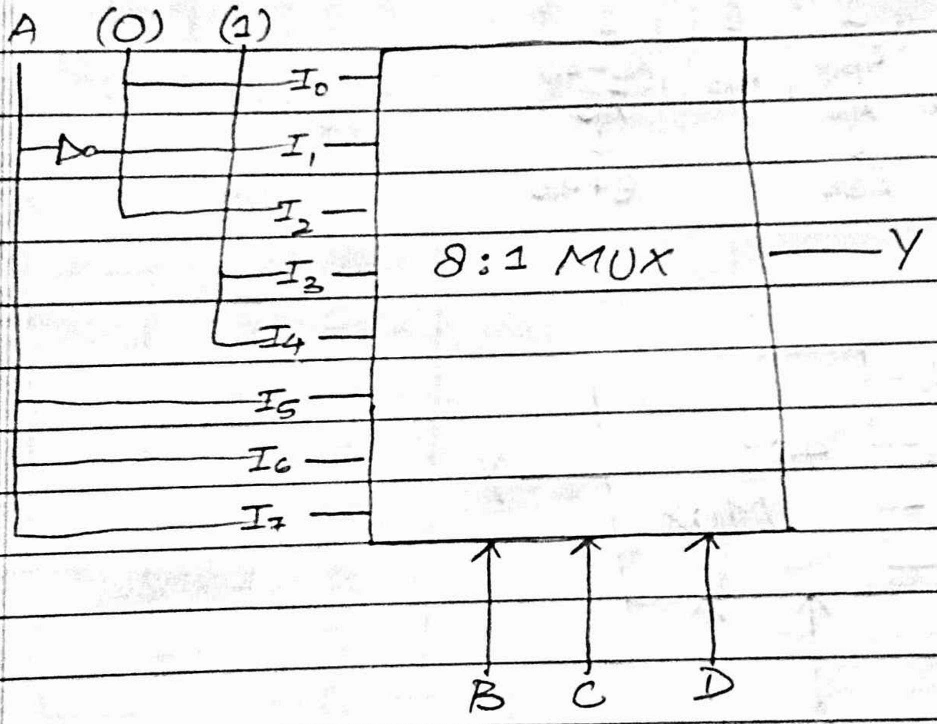


$f(A, B, C, D) = \Sigma m(1, 3, 4, 11, 12, 13, 14, 15)$

Implement using 8:1 MUX

| A \ BCD | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
|---|---|---|---|---|---|---|---|---|
| 0 | $^{0}$ | $1^{1}$ | $^{2}$ | $1^{3}$ | $1^{4}$ | $^{5}$ | $^{6}$ | $^{7}$ |
| 1 | $^{8}$ | $^{9}$ | $^{10}$ | $1^{11}$ | $1^{12}$ | $1^{13}$ | $1^{14}$ | $1^{15}$ |
| | 0 | $\bar{A}$ | 0 | 1 | 1 | A | A | A |

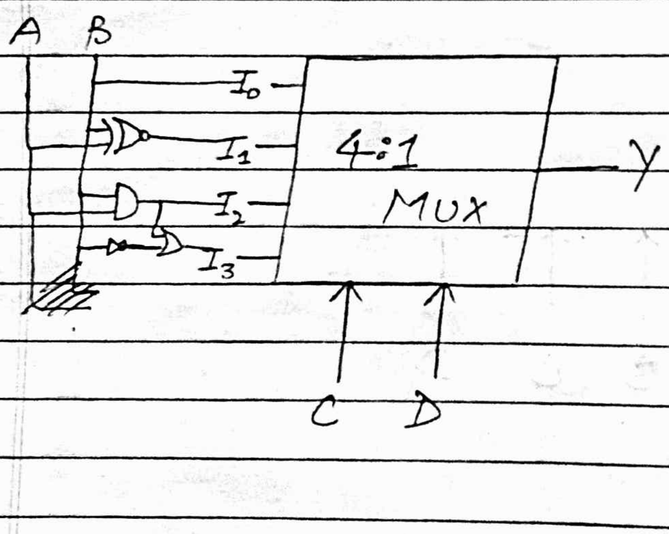
8:1
↓
3 select lines
BCD → select lines
A → input



$f(A,B,C,D) = \sum m(1,3,4,11,12,13,14,15)$ Implement using 4:1 Mux

Select lines → CD
input → AB

| AB \ CD | $I_0$ 00 | $I_1$ 01 | $I_2$ 10 | $I_3$ 11 |
|---|---|---|---|---|
| 00 | [0] | 1 [1] | [2] | 1 [3] |
| 01 | 1 [4] | [5] | [6] | [7] |
| 10 | [8] | [9] | [10] | 1 [11] |
| 11 | 1 [12] | 1 [13] | 1 [14] | 1 [15] |
| | $\bar{A}B + AB = B$ | $\bar{A}\bar{B} + AB \to \overline{A \oplus B}$ | $AB$ | $\bar{A}\bar{B} + A\bar{B} + AB \to \bar{B} + AB$ |



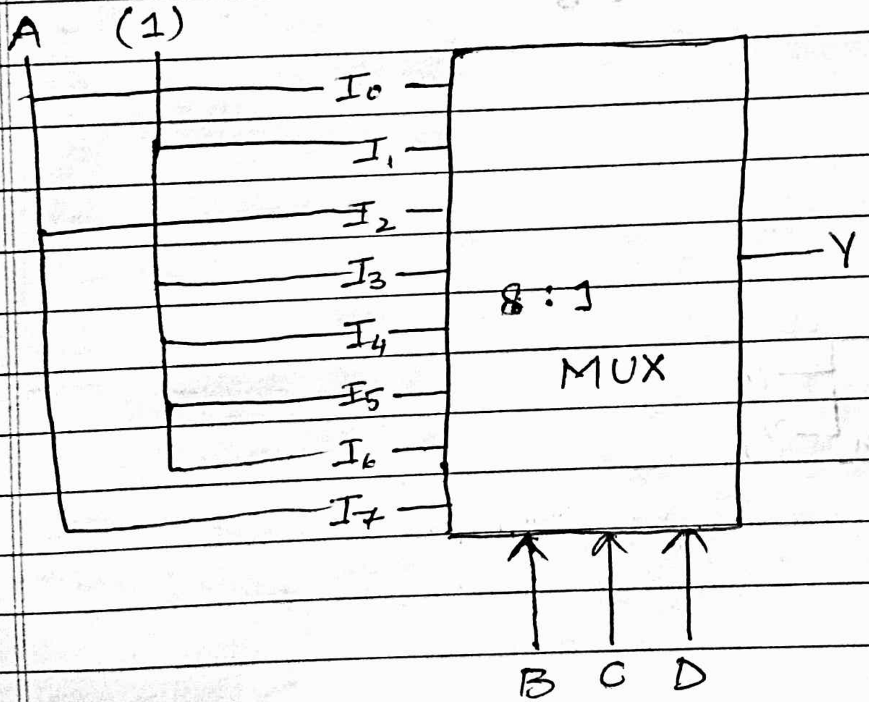
$f(A, B, C, D) = \sum m(1, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$

using 8:1 MUX

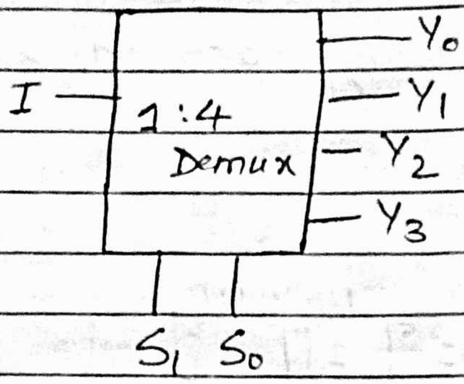
BCD → select lines

A → input

| A \ BCD | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
|---|---|---|---|---|---|---|---|---|
| 0 | [0] | 1 [1] | [2] | 1 [3] | 1 [4] | 1 [5] | 1 [6] | [7] |
| 1 | 1 [8] | 1 [9] | 1 [10] | 1 [11] | 1 [12] | 1 [13] | 1 [14] | 1 [15] |
| | A | 1 | A | 1 | 1 | 1 | 1 | A |



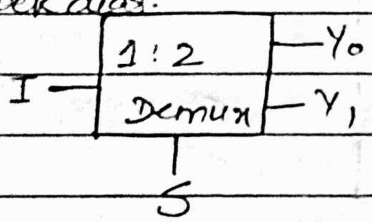
Demux → one to many




Demux → 1:2, 1:4, 1:8, 1:16, 1:32, 1:64

1:2 Demux

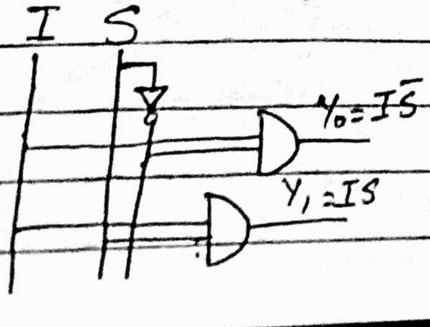
I Block diag.



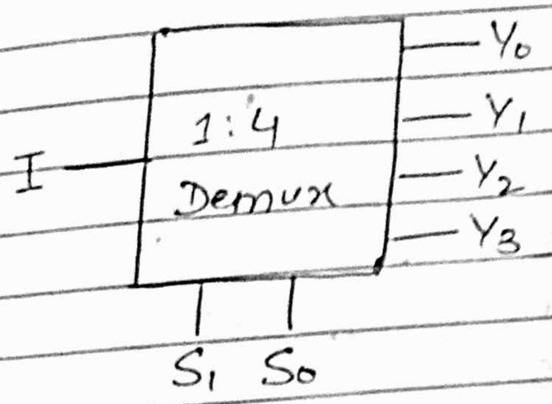

II Truth Table

| I | S | Y |
|---|---|---|
| I | 0 | $Y_0 = I\bar{S}$ |
| I | 1 | $Y_1 = IS$ |

III Exp. – $Y_0 = I\bar{S}$ $Y_1 = IS$



1:4 Demux




Truth Table

| I | $S_1$ | $S_0$ | Y |
|---|---|---|---|
| I | 0 | 0 | $Y_0$ |
| I | 0 | 1 | $Y_1$ |
| I | 1 | 0 | $Y_2$ |
| I | 1 | 1 | $Y_3$ |

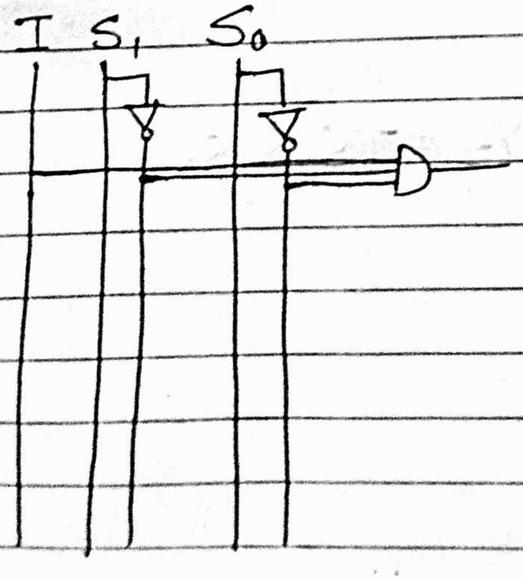
Exp.

$Y_0 = I\bar{S_1}\bar{S_0}$

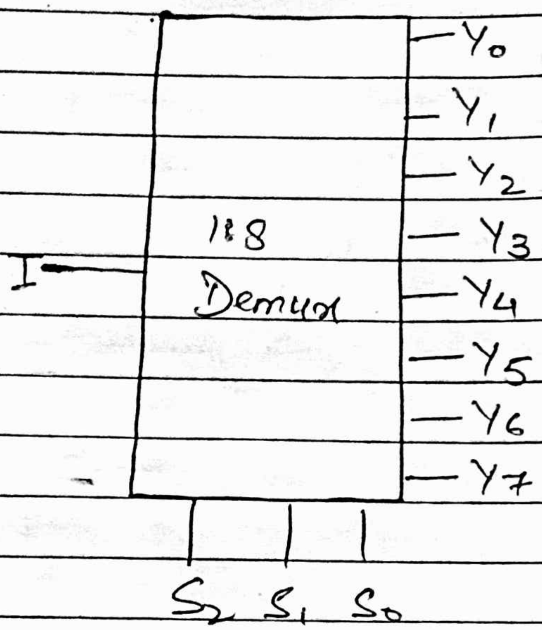
$Y_1 = I\bar{S_1}S_0$

$Y_2 = IS_1\bar{S_0}$

$Y_3 = IS_1S_0$



## 1:8 Demux




| I | $S_2$ | $S_1$ | $S_0$ | Y |
|---|---|---|---|---|
| I | 0 | 0 | 0 | $Y_0$ |
| I | 0 | 0 | 1 | $Y_1$ |
| I | 0 | 1 | 0 | $Y_2$ |
| I | 0 | 1 | 1 | $Y_3$ |
| I | 1 | 0 | 0 | $Y_4$ |
| I | 1 | 0 | 1 | $Y_5$ |
| I | 1 | 1 | 0 | $Y_6$ |
| I | 1 | 1 | 1 | $Y_7$ |

Exp:

$Y_0 = I\bar{S}_2\bar{S}_1\bar{S}_0$

$Y_1 = I\bar{S}_2\bar{S}_1 S_0$

$Y_2 = I\bar{S}_2 S_1 \bar{S}_0$

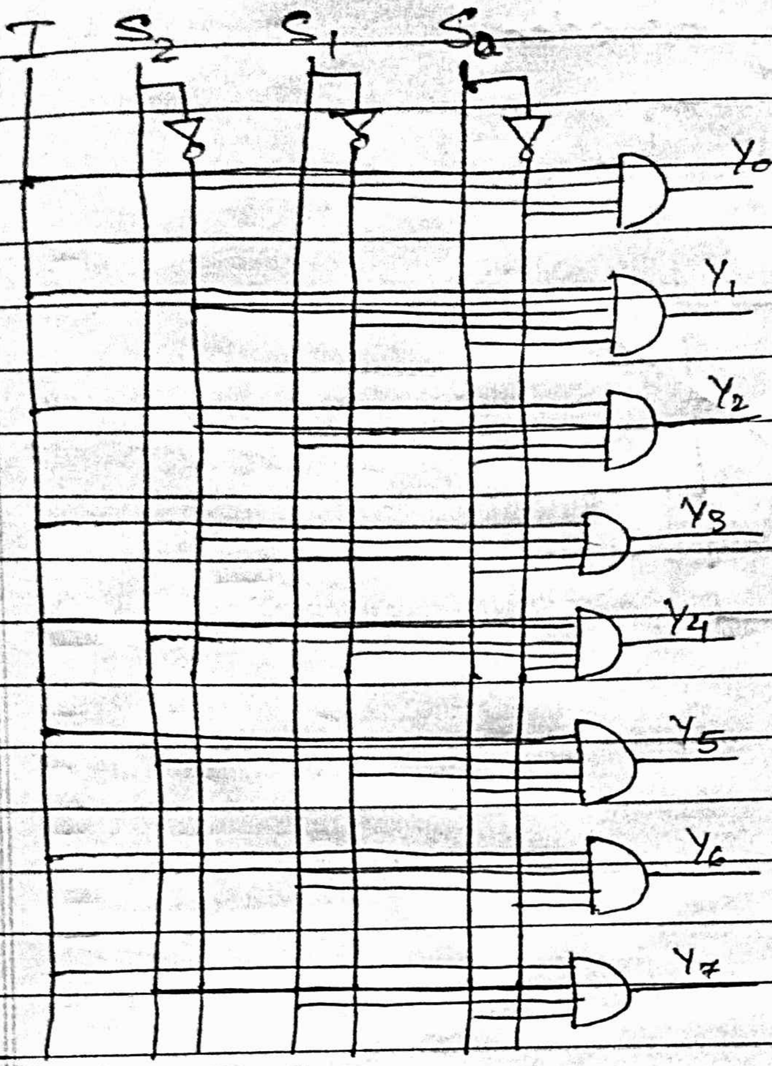
$Y_3 = I\bar{S}_2 S_1 S_0$

$Y_4 = I S_2 \bar{S}_1 \bar{S}_0$

$Y_5 = I S_2 \bar{S}_1 S_0$

$Y_6 = I S_2 S_1 \bar{S}_0$

$Y_7 = I S_2 S_1 S_0$.


I
$S_2$
$S_1$
$S_0$
$Y_0$
$Y_1$
$Y_2$
$Y_3$
$Y_4$
$Y_5$
$Y_6$
$Y_7$

MUX

Many to one

I/P > O/P

Select lines are there

8:1

4:1

2:1

Encoder

Many to many

I/P>O/P

No select lines

8:3

4:2

2:1

Demux

One to many

I/P<O/P

Select lines are there

1:4

1:8

1:16

Decoder

Many to many

I/P<O/P

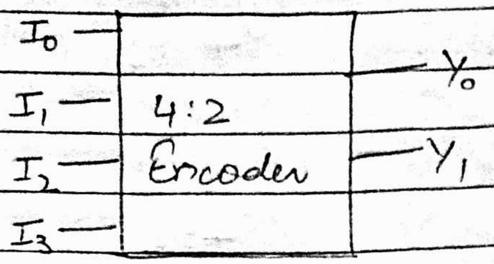
No select lines

2:4

3:8

Encoder [4:2]

I




II Truth Table

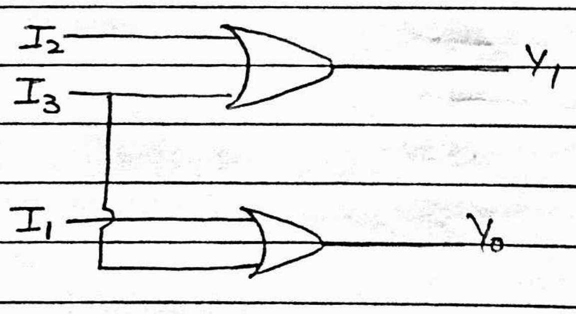
| $I_0$ | $I_1$ | $I_2$ | $I_3$ | $Y_1$ | $Y_0$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |

III Expression

$Y_0 = I_1 + I_3$

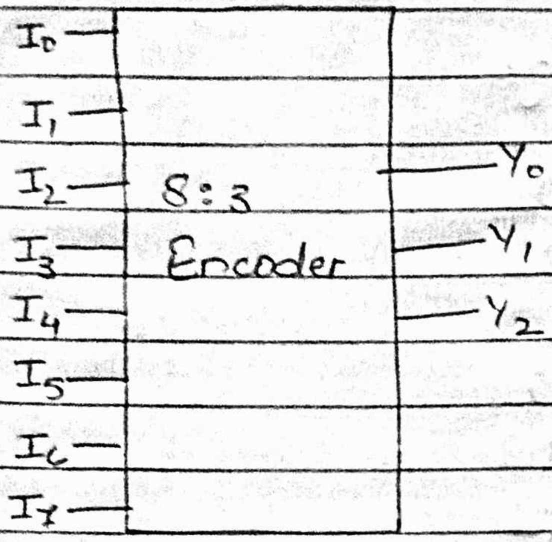
$Y_1 = I_2 + I_3$

IV



## Encoder [8:3]

I




II Truth Table

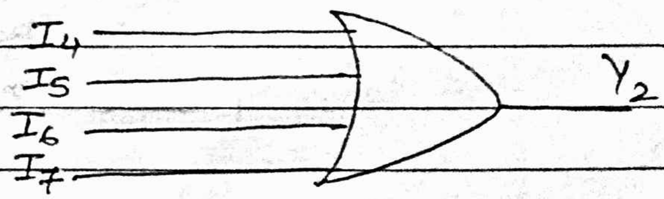
| $I_0$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | $Y_2$ | $Y_1$ | $Y_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

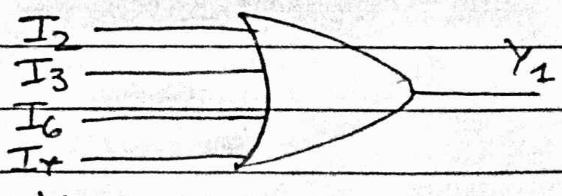
III Expression

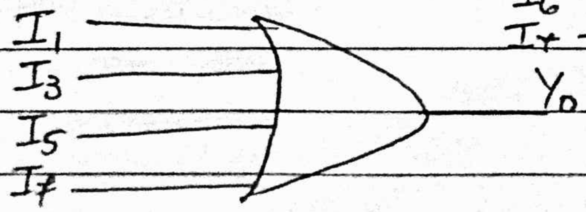
$Y_2 = I_4 + I_5 + I_6 + I_7$

$Y_1 = I_2 + I_3 + I_6 + I_7$

$Y_0 = I_1 + I_3 + I_5 + I_7$



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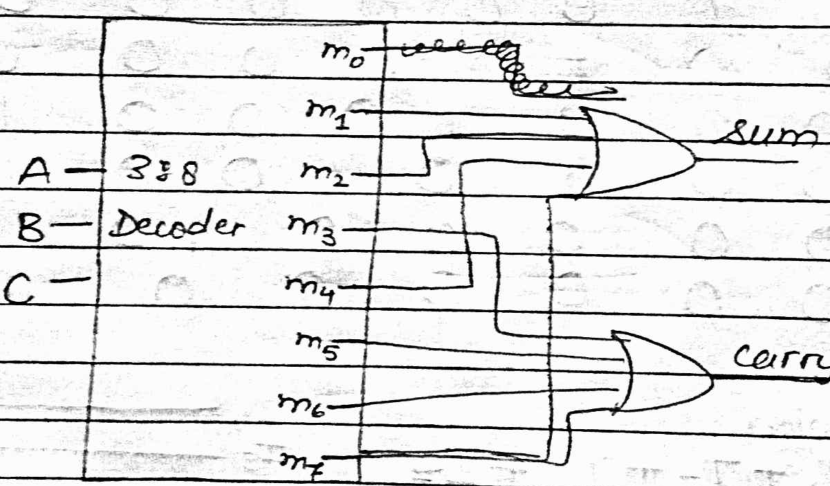
Implement full adder using 3:8 Decoder

| A | B | C | sum | carry |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

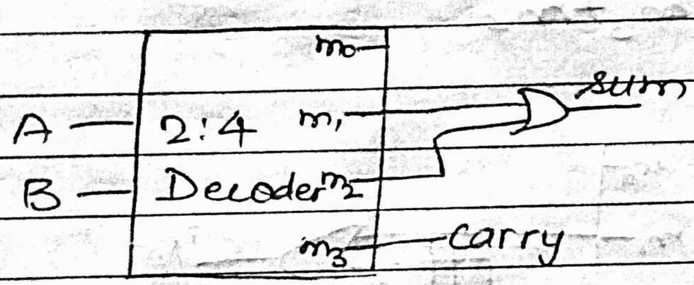
$sum = m_1 + m_2 + m_4 + m_7$
$carry = m_3 + m_5 + m_6 + m_7$



Implement half adder on 2:4 decoder

| A | B | sum | carry |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

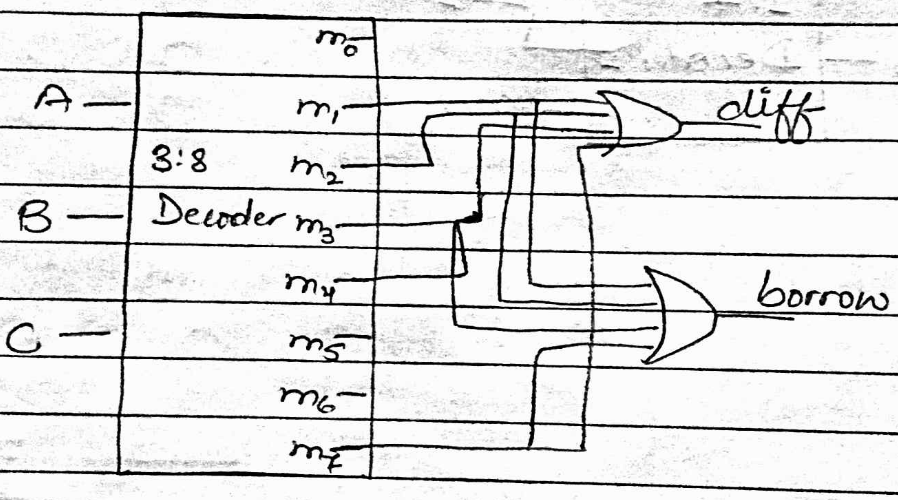
$sum = m_1 + m_2$
$carry = m_3$



Implement full subtractor using 3:8 Decoder

| A | B | C | diff | borrow |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$diff = m_1 + m_2 + m_4 + m_7$
$borrow = m_1 + m_2 + m_3 + m_7$



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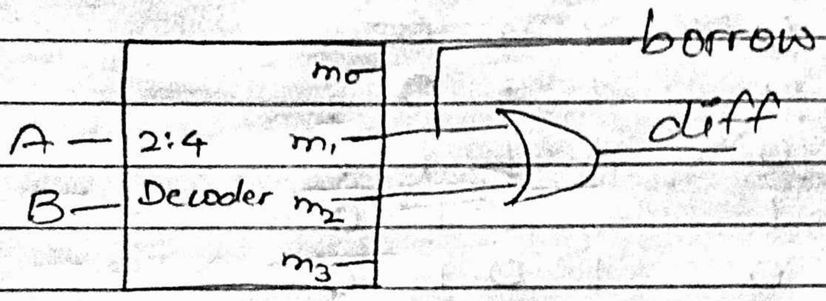
Implement Half subtractor using 2:4 Decoder

| A | B | diff | borrow |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

diff $= m_1 + m_2$

borrow $= m_1$



## Code Converter

### Binary to Gray

I

| $B_0$ | $B_1$ | $B_2$ | $B_3$ | $G_0$ | $G_1$ | $G_2$ | $G_3$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

II $G_0 = \Sigma m\,(8, 9, 10, 11, 12, 13, 14, 15)$

| $B_0B_1 \backslash B_2B_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 12: 1 | 13: 1 | 15: 1 | 14: 1 |
| 10 | 8: 1 | 9: 1 | 11: 1 | 10: 1 |

$\rightarrow B_0$

$G_0 = B_0$

$G_1 = \Sigma m(4, 5, 6, 7, 8, 9, 10, 11)$

| $B_0B_1 \backslash B_2B_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 1 (4) | 1 (5) | 1 (7) | 1 (6) |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 1 (8) | 1 (9) | 1 (11) | 1 (10) |

$G_1 = \bar{B}_0 B_1 + B_0 \bar{B}_1 = B_0 \oplus B_1$

$G_2 = \Sigma m(2, 3, 4, 5, 10, 11, 12, 13)$

| $B_0B_1 \backslash B_2B_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 (3) | 1 (2) |
| 01 | 1 (4) | 1 (5) | 7 | 6 |
| 11 | 1 (12) | 1 (13) | 15 | 14 |
| 10 | 8 | 9 | 1 (11) | 1 (10) |

$G_2 = B_1 \bar{B}_2 + \bar{B}_1 B_2 = B_1 \oplus B_2$

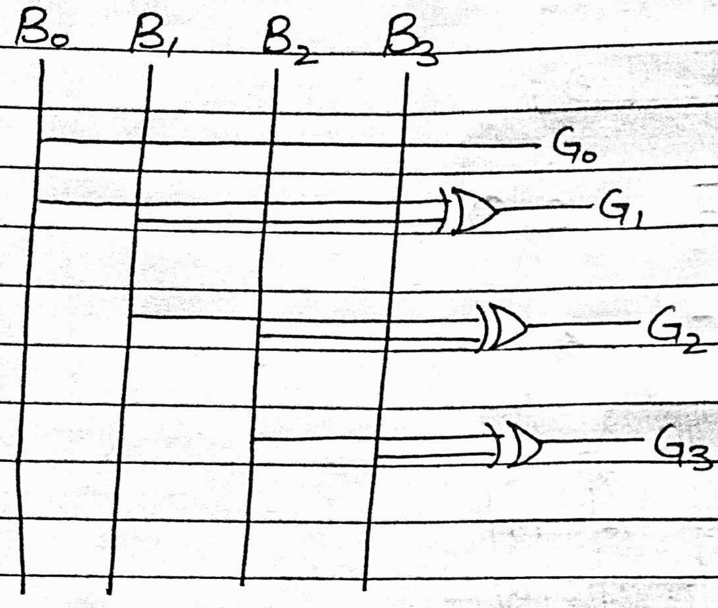
$G_3 = \Sigma m(1, 2, 5, 6, 9, 10, 13, 14)$

| $B_0B_1 \backslash B_2B_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 (1) | 3 | 1 (2) |
| 01 | 4 | 1 (5) | 7 | 1 (6) |
| 11 | 12 | 1 (13) | 15 | 1 (14) |
| 10 | 8 | 1 (9) | 11 | 1 (10) |

$G_3 = \bar{B}_2 B_3 + B_2 \bar{B}_3 = B_2 \oplus B_3$

~~Gray to Binary~~

Realization :




Gray to Binary

| $G_0$ | $G_1$ | $G_2$ | $G_3$ | $B_0$ | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |

$B_0 = \sum m(8, 9, 10, 11, 12, 13, 14, 15)$

| $G_0G_1 \backslash G_2G_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 1 (12) | 1 (13) | 1 (15) | 1 (14) |
| 10 | 1 (8) | 1 (9) | 1 (11) | 1 (10) |

$B_0 = G_0$

$B_1 = \sum m(4, 5, 6, 7, 8, 9, 10, 11)$

| $G_0G_1 \backslash G_2G_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 1 (4) | 1 (5) | 1 (7) | 1 (6) |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 1 (8) | 1 (9) | 1 (11) | 1 (10) |

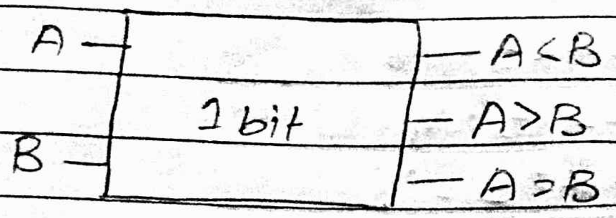
$B_1 = \overline{G_0}G_1 + G_0\overline{G_1} = G_0 \oplus G_1$

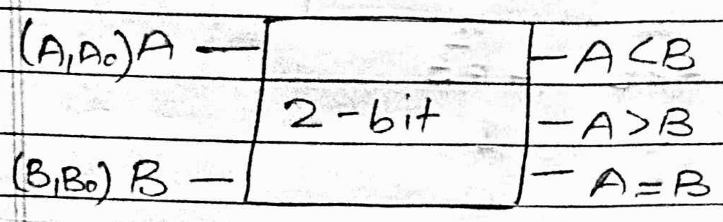
$B_2 = \sum m(2, 3, 4, 5, 8, 9, 14, 15)$

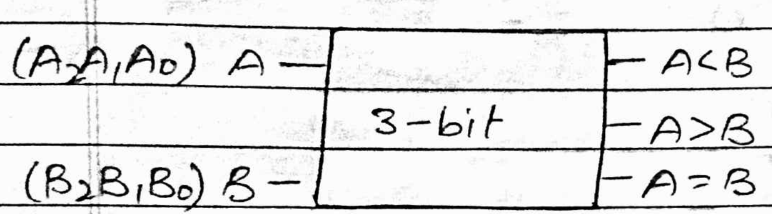
| $G_0G_1 \backslash G_2G_3$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 (3) | 1 (2) |
| 01 | 1 (4) | 1 (5) | 7 | 6 |
| 11 | 12 | 13 | 1 (15) | 1 (14) |
| 10 | 1 (8) | 1 (9) | 11 | 10 |

$$B_2 = \overline{G_0}\,\overline{G_1}G_2 + \overline{G_0}G_1\overline{G_2} + G_0G_1G_2 + G_0\overline{G_1}\,\overline{G_2}$$
$$= \overline{G_0}(\overline{G_1}G_2 + G_1\overline{G_2}) + G_0(G_1G_2 + \overline{G_1}\,\overline{G_2})$$
$$= \overline{G_0}(G_1 \oplus G_2) + G_0(\overline{G_1 \oplus G_2})$$

Comparator [2 inputs 3 outputs]
↳ [1 would be high]



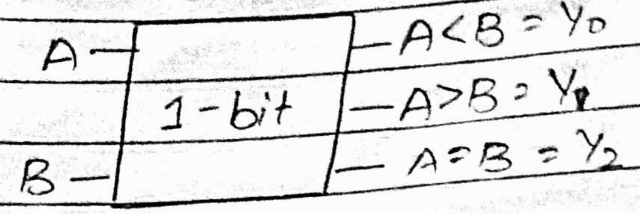




Types of comparator
- Identity → Two values [only one input] one output
- Magnitude → Two input, 3 output

# 1-Bit Comparator

I.




II. Truth Table

| | A | B | $Y_0$ A<B | $Y_1$ A>B | $Y_2$ A=B |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 |
| 3 | 1 | 1 | 0 | 0 | 1 |

III. $Y_0 = \bar{A}B$

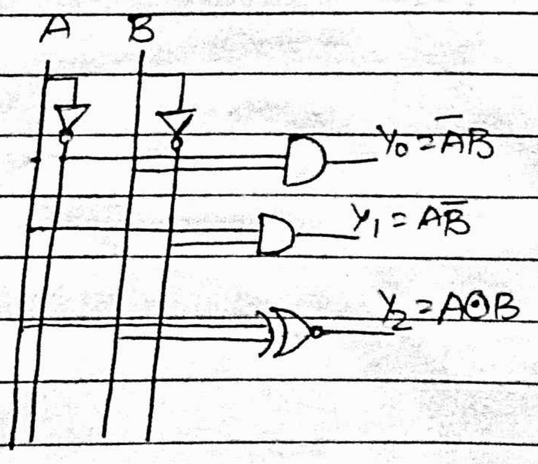
| A \ B | 0 | 1 |
|---|---|---|
| 0 | | 1 → $\bar{A}B$ |
| 1 | | |

$Y_1 = A\bar{B}$

$Y_2 = \bar{A}\bar{B} + AB = \overline{A \oplus B} = A \odot B$

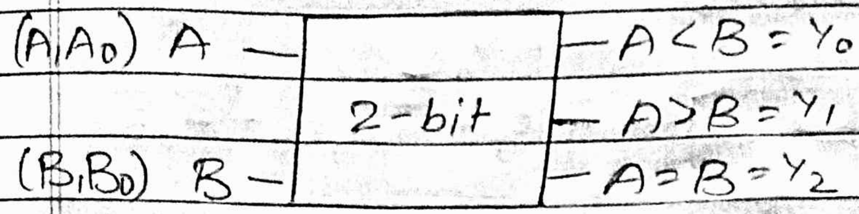
| A \ B | 0 | 1 |
|---|---|---|
| 0 | 1 | |
| 1 | | 1 |

IV.



2-bit

I




II Truth Table

| $A_1$ | $A_0$ | $B_1$ | $B_0$ | $Y_0$ $A<B$ | $Y_1$ $A>B$ | $Y_2$ $A=B$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 |

III $Y_0 = \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0$

| $A_1A_0$ \ $B_1B_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | 1 | 1 |
| 01 | | | 1 | 1 |
| 11 | | | | |
| 10 | | | 1 | |

$Y_0 \rightarrow \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0$

$Y_1 \rightarrow$

| $A_1A_0$ \ $B_1B_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | 1 | | | |
| 11 | 1 | 1 | | 1 |
| 10 | 1 | 1 | | |

$Y_1 \rightarrow$

$A_1 \bar{B}_1 + A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0$

$Y_2 \rightarrow$

| $A_1A_0$ \ $B_1B_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | | | |
| 01 | | 1 | | |
| 11 | | | 1 | |
| 10 | | | | 1 |

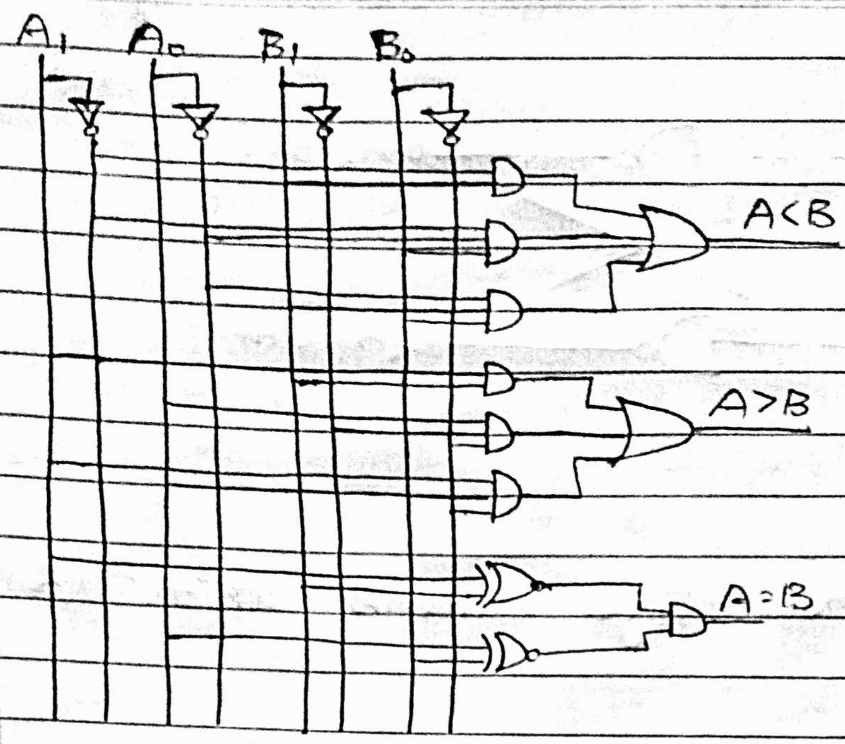
$\bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + A_1 A_0 B_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0$

$\bar{A}_1 \bar{B}_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0) + A_1 B_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0)$

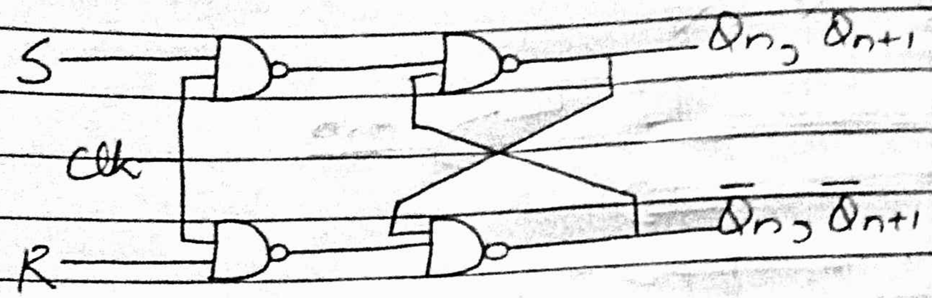
$(\bar{A}_1 \bar{B}_1 + A_1 B_1)(\bar{A}_0 \bar{B}_0 + A_0 B_0)$

$(A_1 \odot B_1)(A_0 \odot B_0)$

IV



## Flip flops SR




→ latches are basic storage elements which operates at signal levels

→

Characteristic Table

| Clk | S | R | $Q_n$ | $Q_{n+1}$ | |
|---|---|---|---|---|---|
| ↑ | 0 | 0 | 0 | 0 | No change |
| ↑ | 0 | 0 | 1 | 1 | |
| ↑ | 0 | 1 | 0 | 0 | Reset |
| ↑ | 0 | 1 | 1 | 0 | |
| ↑ | 1 | 0 | 0 | 1 | Set |
| ↑ | 1 | 0 | 1 | 1 | |
| ↑ | 1 | 1 | 0 | x | Invalid Case |
| ↑ | 1 | 1 | 1 | x | |

Excitation table

| $Q_n$ | $Q_{n+1}$ | S R |
|---|---|---|
| 0 | 0 | 0 0<br>0 1 |
| 0 | 1 | 1 0 |
| 1 | 0 | 0 1 |
| 1 | 1 | 0 0<br>1 0 |

| S | R |
|---|---|
| 0 | x |
| 1 | 0 |
| 0 | 1 |
| x | 0 |

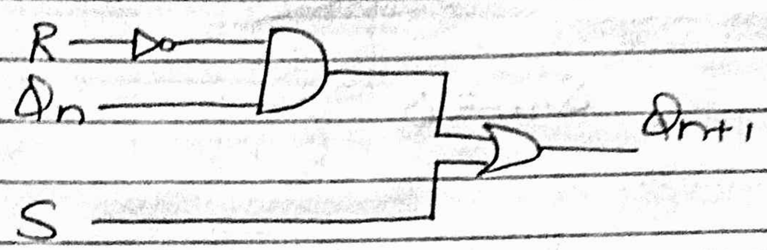
| $Q_n$ | $Q_{n+1}$ | S | R |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |

Exp

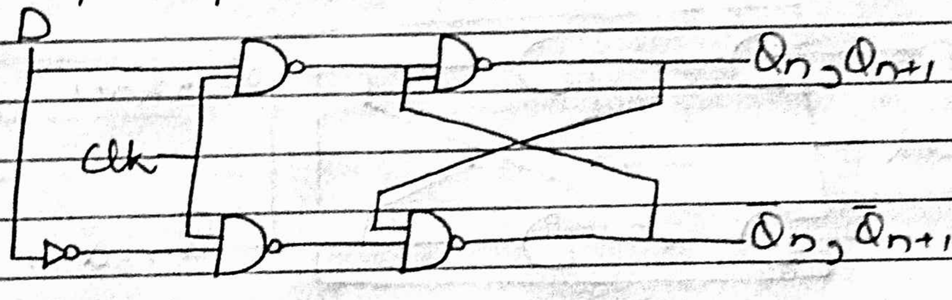
| S \ $RQ_n$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | |
| 1 | 1 | 1 | X | X |

$$Q_{n+1} = S + \bar{R}Q_n$$

Realization




D-Flip Flop




Characteristic Table

| Clk | D | $Q_n$ | $Q_{n+1}$ | |
|---|---|---|---|---|
| ↑ | 0 | 0 | 0 | Reset |
| ↑ | 0 | 1 | 0 | |
| ↑ | 1 | 0 | 1 | Set |
| ↑ | 1 | 1 | 1 | |

## Excitation Table

| $Q_n$ | $Q_{n+1}$ | D |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

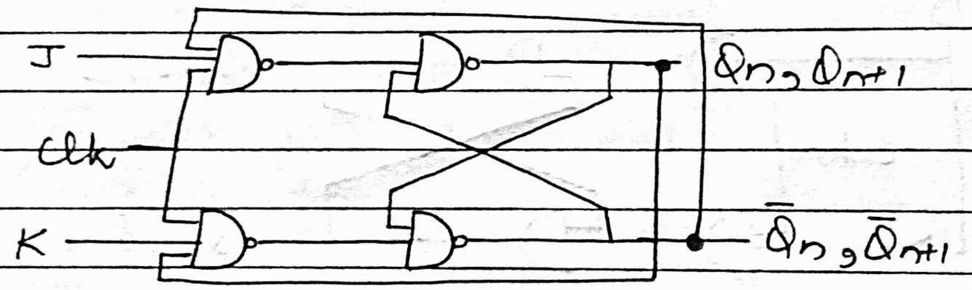
## Exp

| D \ $Q_n$ | 0 | 1 |
|---|---|---|
| 0 | | |
| 1 | 1 | 1 |

$Q_{n+1} = D$

$D \longrightarrow Q_{n+1}$

## JK - Flip Flop




## Characteristic Table

| Clk | J | K | $Q_n$ | $Q_{n+1}$ | |
|---|---|---|---|---|---|
| ↑ | 0 | 0 | 0 | 0 | No change |
| ↑ | 0 | 0 | 1 | 1 | |
| ↑ | 0 | 1 | 0 | 0 | Reset |
| ↑ | 0 | 1 | 1 | 0 | |
| ↑ | 1 | 0 | 0 | 1 | Set |
| ↑ | 1 | 0 | 1 | 1 | |
| ↑ | 1 | 1 | 0 | 1 | Toggle |
| ↑ | 1 | 1 | 1 | 0 | |

# Excitation Table

| $Q_n$ | $Q_{n+1}$ | J | K | J | K |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | X |
| | | 0 | 1 | | |
| 0 | 1 | 1 | 0 | 1 | X |
| | | 1 | 1 | | |
| 1 | 0 | 0 | 1 | X | 1 |
| | | 1 | 1 | | |
| 1 | 1 | 0 | 0 | X | 0 |
| | | 1 | 0 | | |

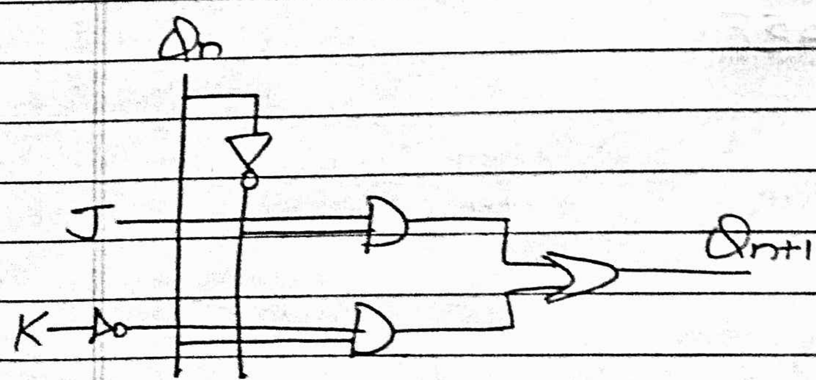
| $Q_n$ | $Q_{n+1}$ | J | K |
|---|---|---|---|
| 0 | 0 | 0 | x |
| 0 | 1 | 1 | x |
| 1 | 0 | x | 1 |
| 1 | 1 | x | 0 |

Exp

| J \ K $Q_n$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | |
| 1 | 1 | 1 | | 1 |

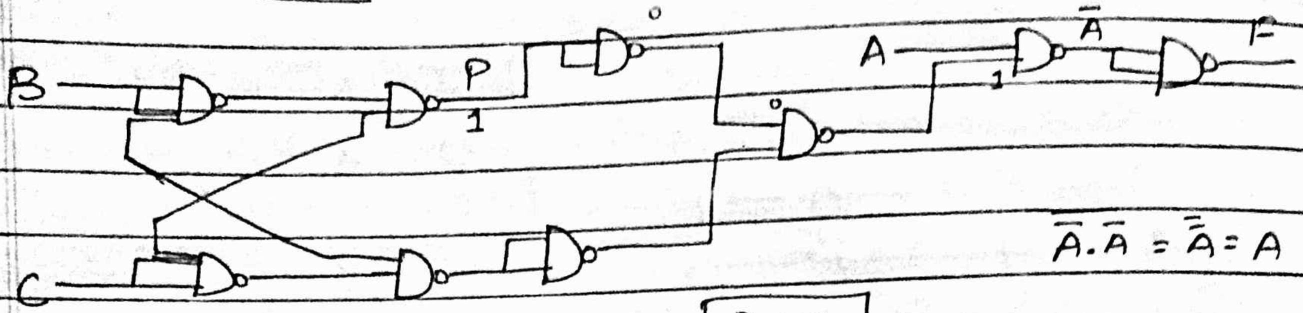
| J \ K $Q_n$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | |
| 1 | 1 | 1 | | 1 |

$$Q_{n+1} = \overline{K}Q_n + J\overline{Q}_n$$



## Practice Questions

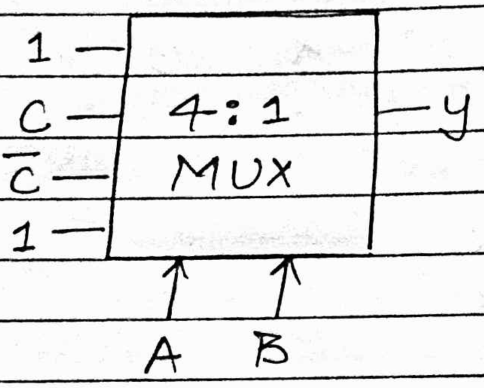
Q1




$\overline{A}\cdot\overline{A} = \overline{\overline{A}} = A$

$\boxed{F = A}$

Q2




| I | A | B | Y |
|---|---|---|---|
| 1 | 0 | 0 | $\bar{A}\bar{B}$ |
| C | 0 | 1 | $\bar{A}BC$ |
| $\bar{C}$ | 1 | 0 | $A\bar{B}\bar{C}$ |
| 1 | 1 | 1 | $AB$ |

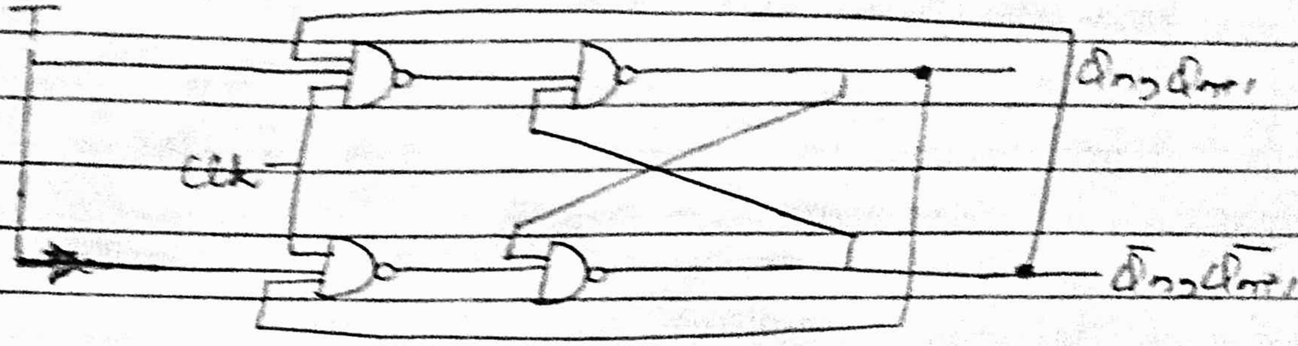
$\bar{A}\bar{B} + AB + \bar{A}BC + A\bar{B}\bar{C}$

$\bar{A}(\bar{B} + BC) + AB + A\bar{B}\bar{C}$

$\bar{A}(\bar{B} + B)(\bar{B} + C) + AB + A\bar{B}\bar{C}$

$\bar{A}\bar{B} + \bar{A}C + AB + A\bar{B}\bar{C}$

## T-Flip Flop




### Characteristic Table

| Clk | T | $Q_n$ | $Q_{n+1}$ | |
|---|---|---|---|---|
| ↑ | 0 | 0 | 0 | No change |
| ↑ | 0 | 1 | 1 | |
| ↑ | 1 | 0 | 1 | Toggle |
| ↑ | 1 | 1 | 0 | |

### Excitation Table

| $Q_n$ | $Q_{n+1}$ | T |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**Exp**

| T \ $Q_n$ | 0 | 1 |
|---|---|---|
| 0 | [0] | 1 [1] |
| 1 | 1 [2] | [3] |

$$Q_{n+1} = \bar{T}Q_n + T\bar{Q}_n$$

### Realization

# Conversion of Flip Flop

## JK to D

① Given → Excitation table

JK

| $Q_n$ | $Q_{n+1}$ | J | K |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

② Req → Characteristic table

D

| D | $Q_n$ | $Q_{n+1}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

③ Combine step ① and ②

| D | $Q_n$ | $Q_{n+1}$ | J | K |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X |
| 0 | 1 | 0 | X | 1 |
| 1 | 0 | 1 | 1 | X |
| 1 | 1 | 1 | X | 0 |

④ Note: $Q_{n+1}$ don't use after step ③

| D \ $Q_n$ | 0 | 1 |
|---|---|---|
| 0 | 0 | X |
| 1 | 1 | X |

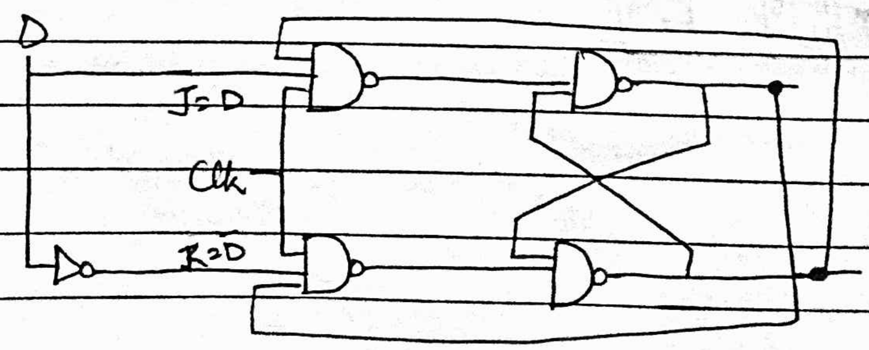
→ D

$J = D$

| D \ $Q_n$ | 0 | 1 |
|---|---|---|
| 0 | X | 1 |
| 1 | X | |

→ $\overline{D}$

$K = \overline{D}$



D to JK

Given

| $Q_n$ | $Q_{n+1}$ | D |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Req.

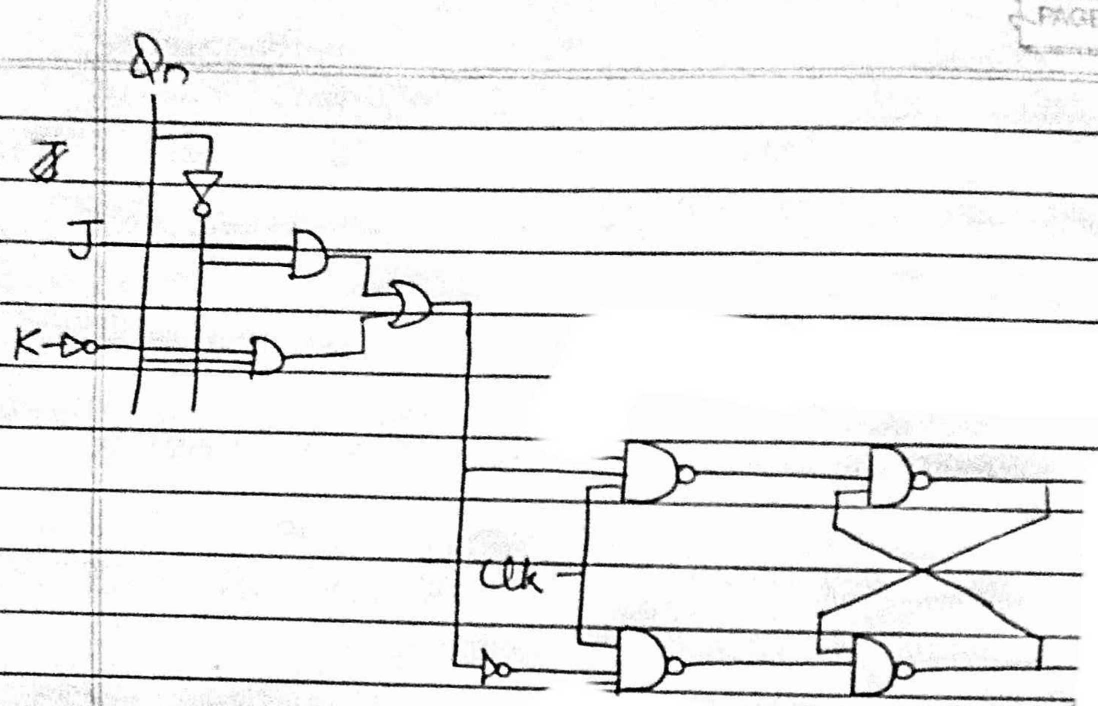
| J | K | $Q_n$ | $Q_{n+1}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Combine

| J | K | $Q_n$ | $Q_{n+1}$ | D |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |

| J \ $KQ_n$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | (0) | 1 (1) | (3) | (2) |
| 1 | 1 (4) | 1 (5) | (7) | 1 (6) |

$D = \bar{K}Q_n + J\bar{Q}_n$


Qn
J
K
Clk

DATE / 20
PAGE No

SR to JK

SR Excitation Table

| $Q_n$ | $Q_{n+1}$ | S | R |
|---|---|---|---|
| 0 | 0 | 0 | x |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | x | 0 |

JK Charac Table

| Clk | J | K | $Q_n$ | $Q_{n+1}$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

Combine —

| J | K | $Q_n$ | $Q_{n+1}$ | S | R |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | x |
| 0 | 0 | 1 | 1 | x | 0 |
| 0 | 1 | 0 | 0 | 0 | x |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | x | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |

Exp for S:-

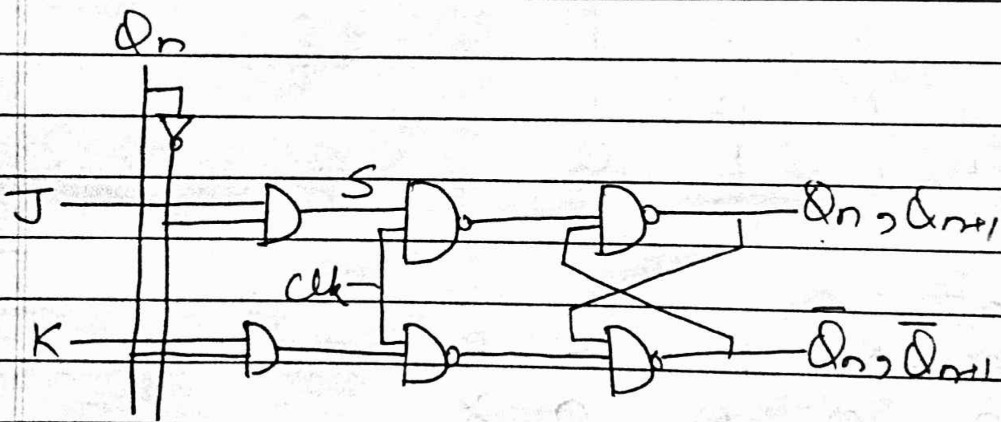
| J \ $KQ_n$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | X | | |
| 1 | 1 | X | | 1 |

$S = J\overline{Q}_n$

Exp for R:-

| J \ $KQ_n$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | X | | 1 | X |
| 1 | | | 1 | |

$R = KQ_n$



## SR to D

SR Excitation Table

| $Q_n$ | $Q_{n+1}$ | S | R |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |

D Charac Table

| Clk | D | $Q_n$ | $Q_{n+1}$ |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Combining

| D | $Q_n$ | $Q_{n+1}$ | S | R |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | X | 0 |

Exp for D

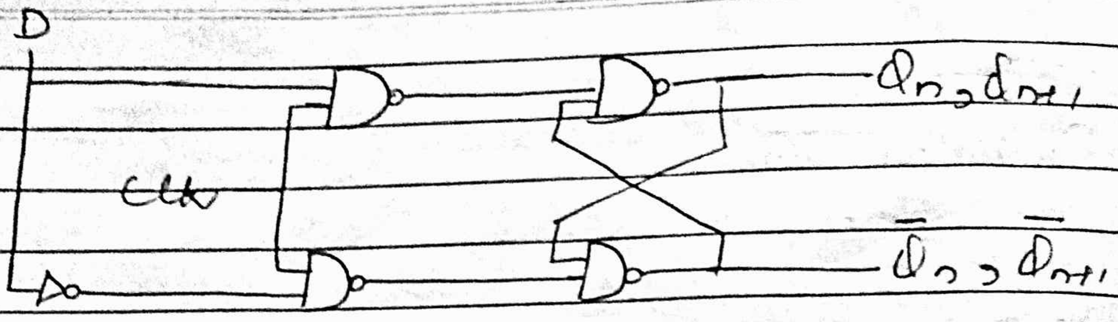
| D \ $Q_n$ | 0 | 1 |
|---|---|---|
| 0 | [0] | [1] |
| 1 | 1 [2] | X [3] |

$S = D$

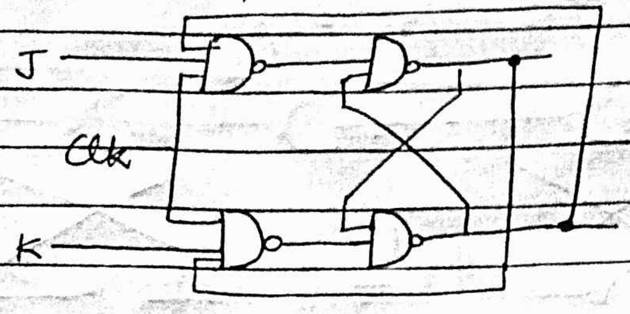
| D \ $Q_n$ | 0 | 1 |
|---|---|---|
| 0 | X [0] | 1 [1] |
| 1 | [2] | [3] |

—1

$R = \overline{D}$


D
Clk
Qn, Qn+1
Q̄n, Q̄n+1

Input to be processed as output → propogation delay.




Race Around Condition

J = 1  K = 1  Clk → ~~+ve~~ +ve edge level trigerred

Removing race around condition

① $T_{pd} < T_{pw}$ → Race around condition will occur

② J = 1  K = 1  clk = +ve level trigerred

① If $T_{pw} < T_{pd}$

② By using Master Slave JK flip flop

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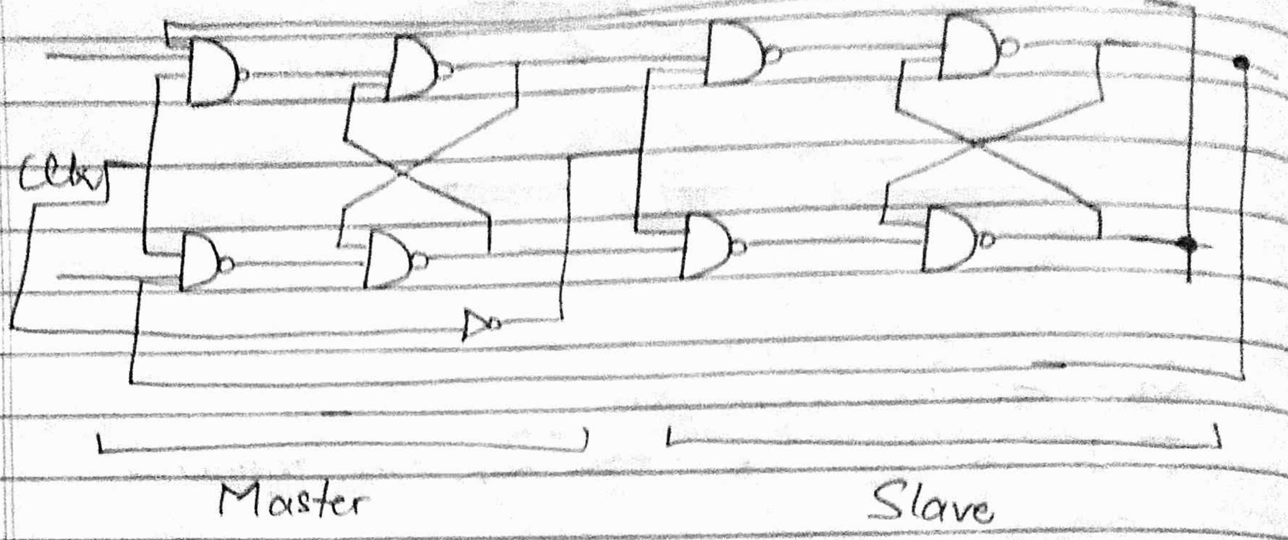
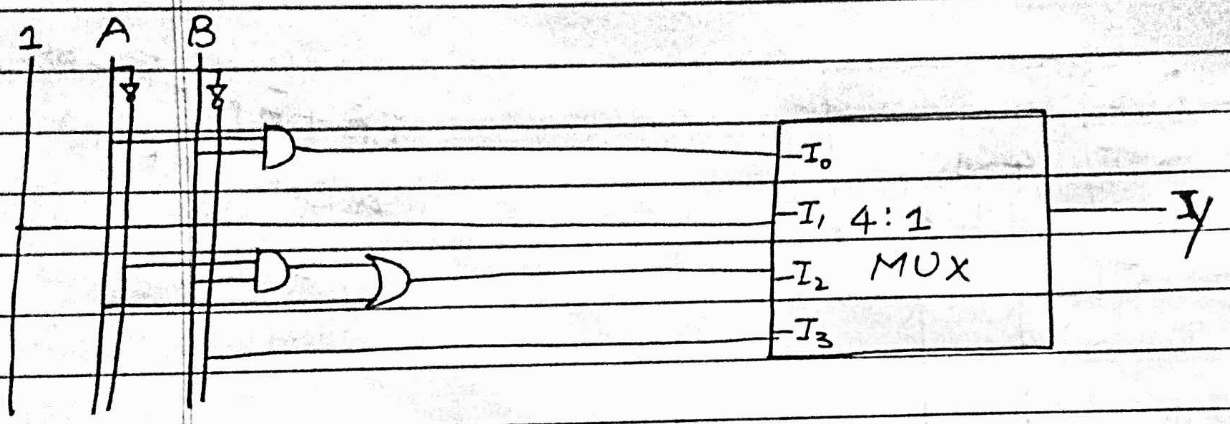
# Master Slave JK Flip Flop



# Practice Ques

$f(A,B,C,D) = \sum m(1,3,5,6,9,10,11,12,13,14)$ using 4:1 MUX

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| AB \ CD | 00 | 01 | 10 | 11 |
| 00 | 0 | 1 [1] | 2 | 1 [3] |
| 01 | 4 | 1 [5] | 1 [6] | 7 |
| 10 | 8 | 1 [9] | 1 [10] | 1 [11] |
| 11 | 1 [12] | 1 [13] | 1 [14] | 15 |
| | $AB$ | $1$ | $A + \bar{A}B$ | $\bar{B}$ |



$\Sigma m(1, 3, 5, 6, 9, 10, 11, 12, 13, 14)$ using 8:1 MUX

| A | $I_0$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|---|---|---|---|---|---|---|---|---|
| 0 | ${}^{0}$ | $1\,{}^{1}$ | ${}^{2}$ | $1\,{}^{3}$ | ${}^{4}$ | $1\,{}^{5}$ | $1\,{}^{6}$ | ${}^{7}$ |
| 1 | ${}^{8}$ | $1\,{}^{9}$ | $1\,{}^{10}$ | $1\,{}^{11}$ | $1\,{}^{12}$ | $1\,{}^{13}$ | $1\,{}^{14}$ | ${}^{15}$ |
| | 0 | 1 | A | 1 | A | 1 | 1 | 0 |